Zurich Research Laboratory

An Efficient Anonymous Credentials System

idemix

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Outline

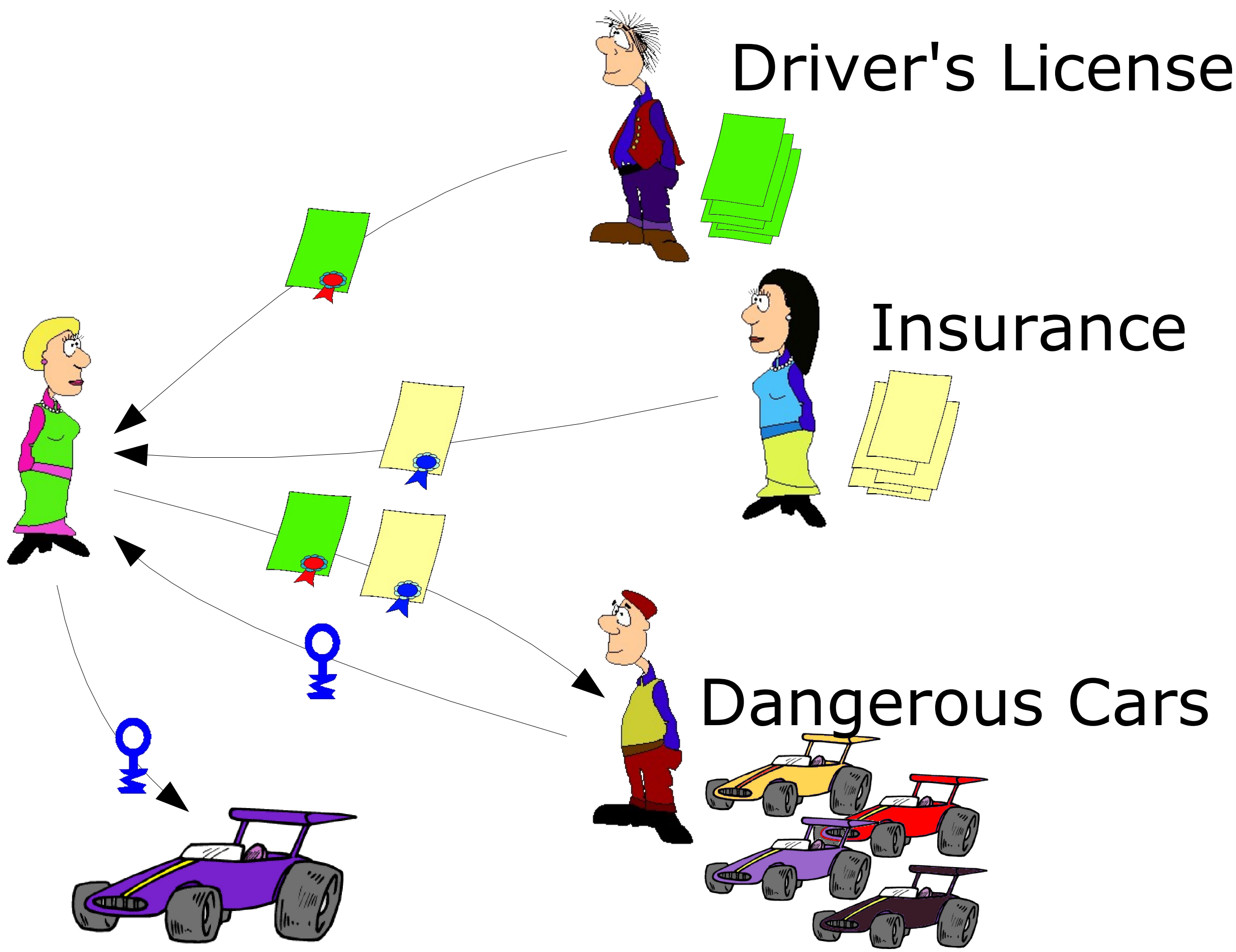
I. Requirements of Anonymous Credential System

II. Abstract Solution

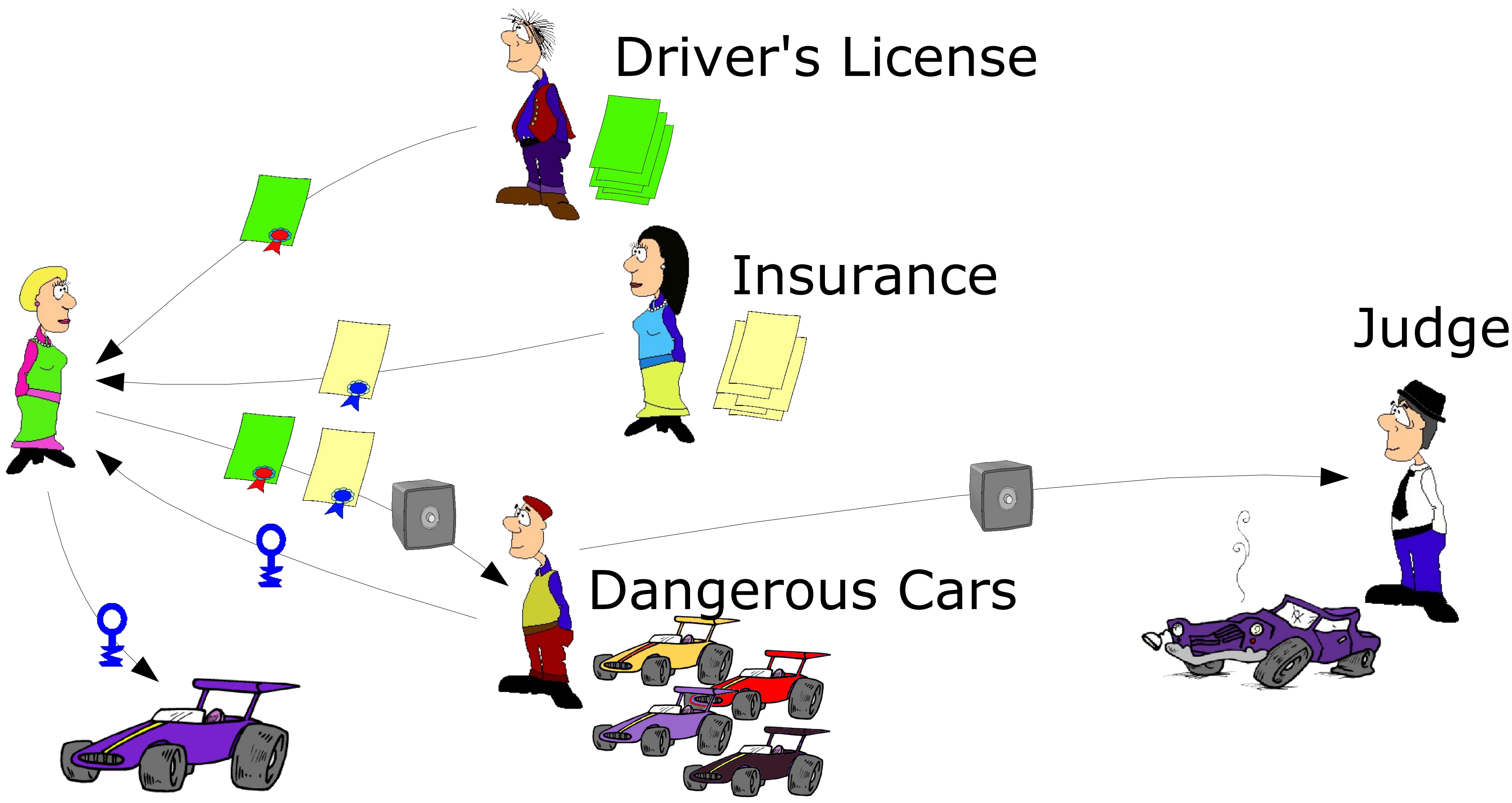
III. The Technical Bit

- Signature Scheme
- Commitments and Proof Protocols
- Encryption Scheme

The Problem: Pseudonym System [Chaum85]

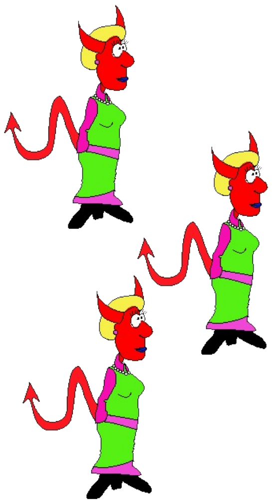


The Problem, Even Larger: Extended Pseudonym System



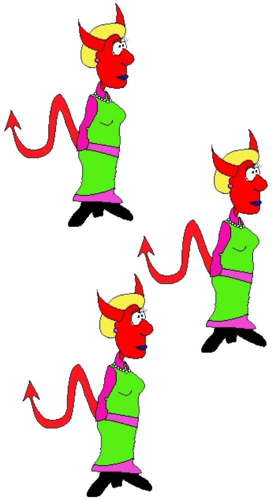
Basic Requirements of Pseudonym System

- Protection of user's privacy
  - anonymity
  - unlinkeability (multi-use)



- Unforgeability of credentials
- Consistency of credentials (no pooling)

Extra Requirements of Pseudonym System



- Sharing of credentials
- Anonymity revocation
  - local
  - global

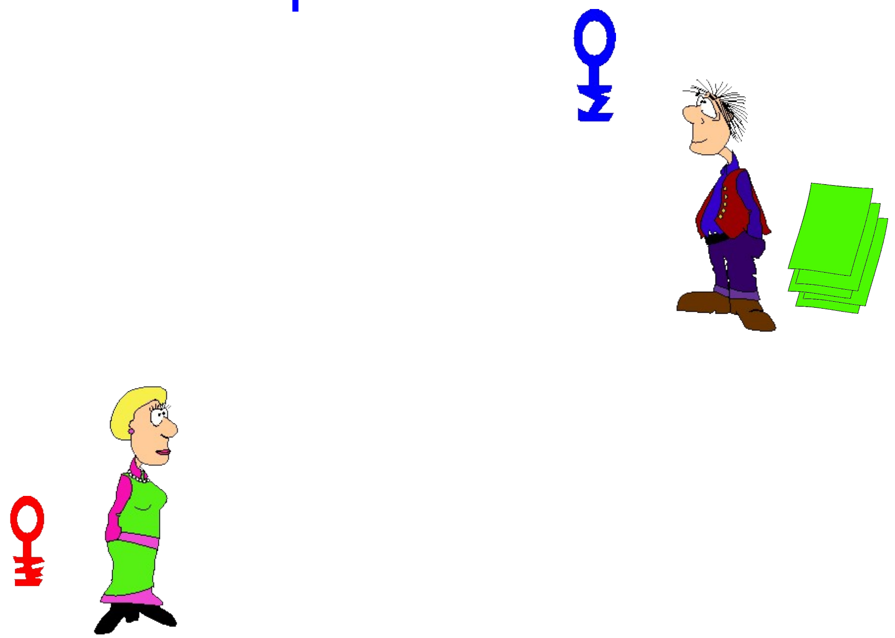
- Revocation of credentials
- Encoding of attributes
- One-show credential (e-cash)
  - off-line & on-line
- k-spendable credentials
- ......

Some History

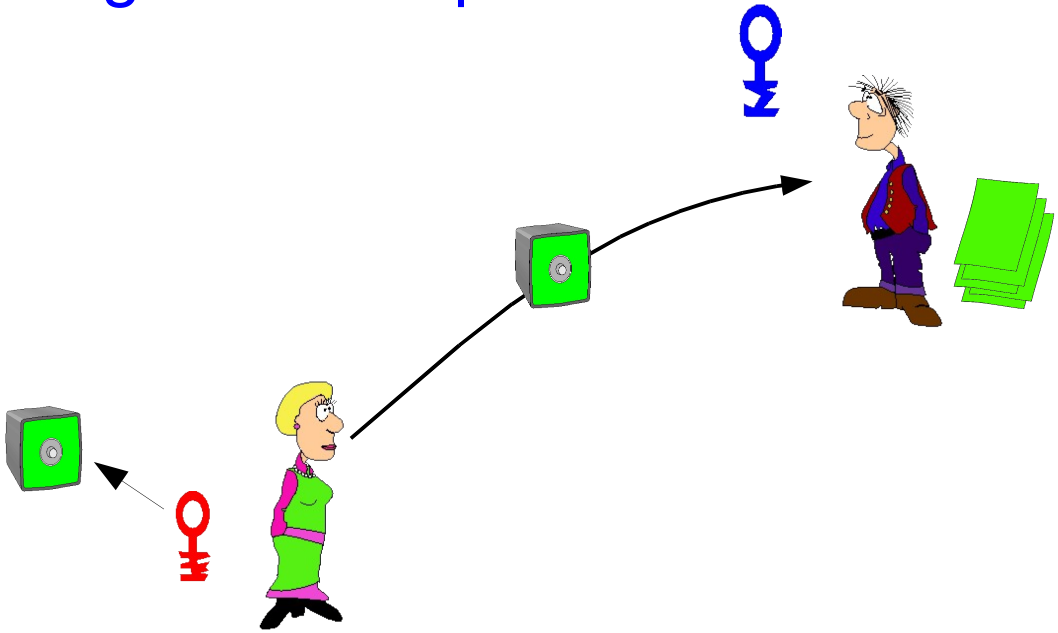
- Chaum ’85: introduced scenario
- Chaum & Evertse ’87: solution based on a semi-trusted party
- Damgård ’90: theoretical solution
- Brands ’95-’99: one-show credentials with different attributes
- LRSW ’99: practical solution for one-show credentials
- Camenisch-Lysyanskaya '00: efficient multi-show w/ attributes
- Verheulen '01: bi-linear map multi-show
- Camenisch-Lysyanskaya '04: Discrete log based.

Special cases: e-cash, group signatures, identity escrow

Proving Ownership Solution



Proving Ownership Solution



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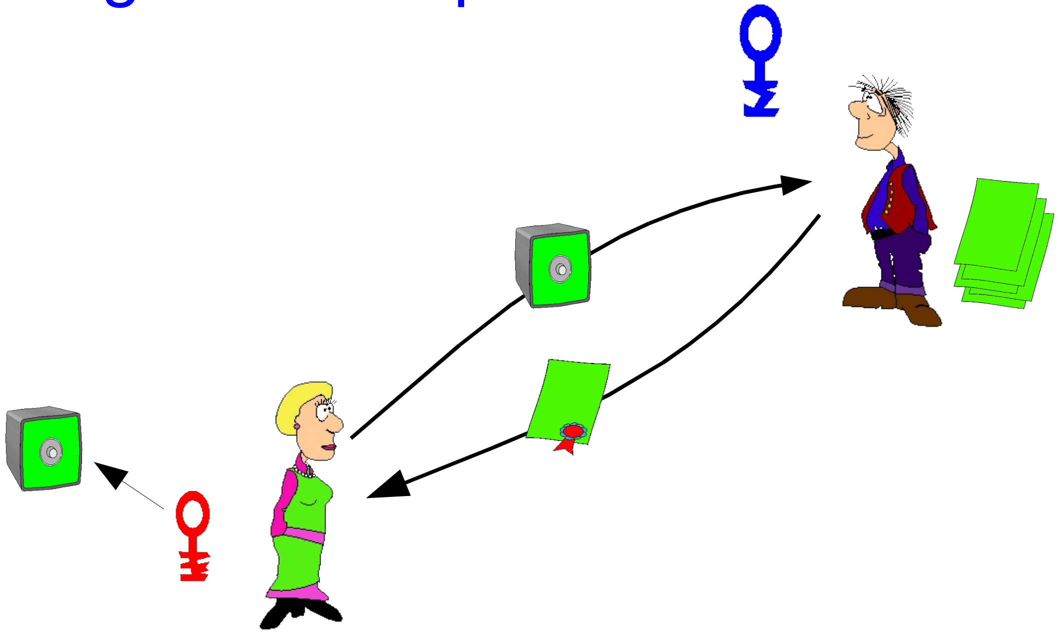

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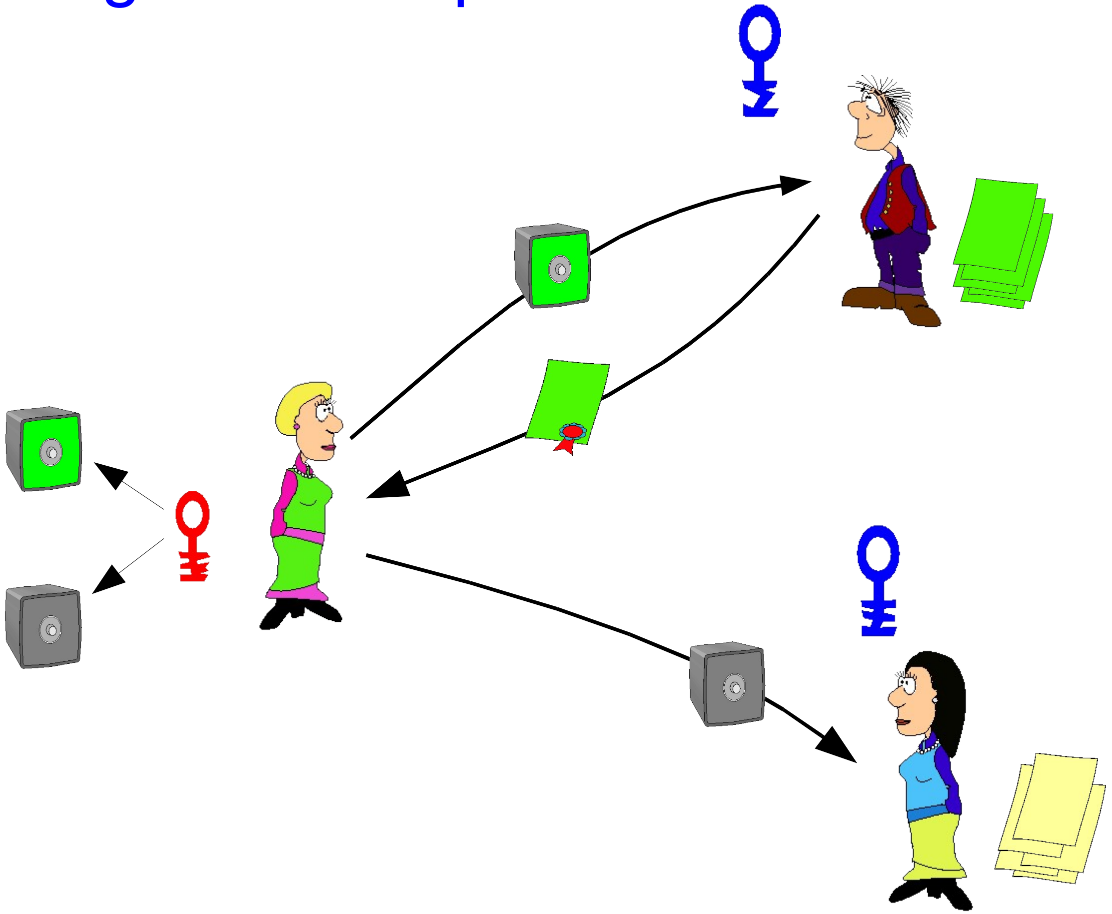

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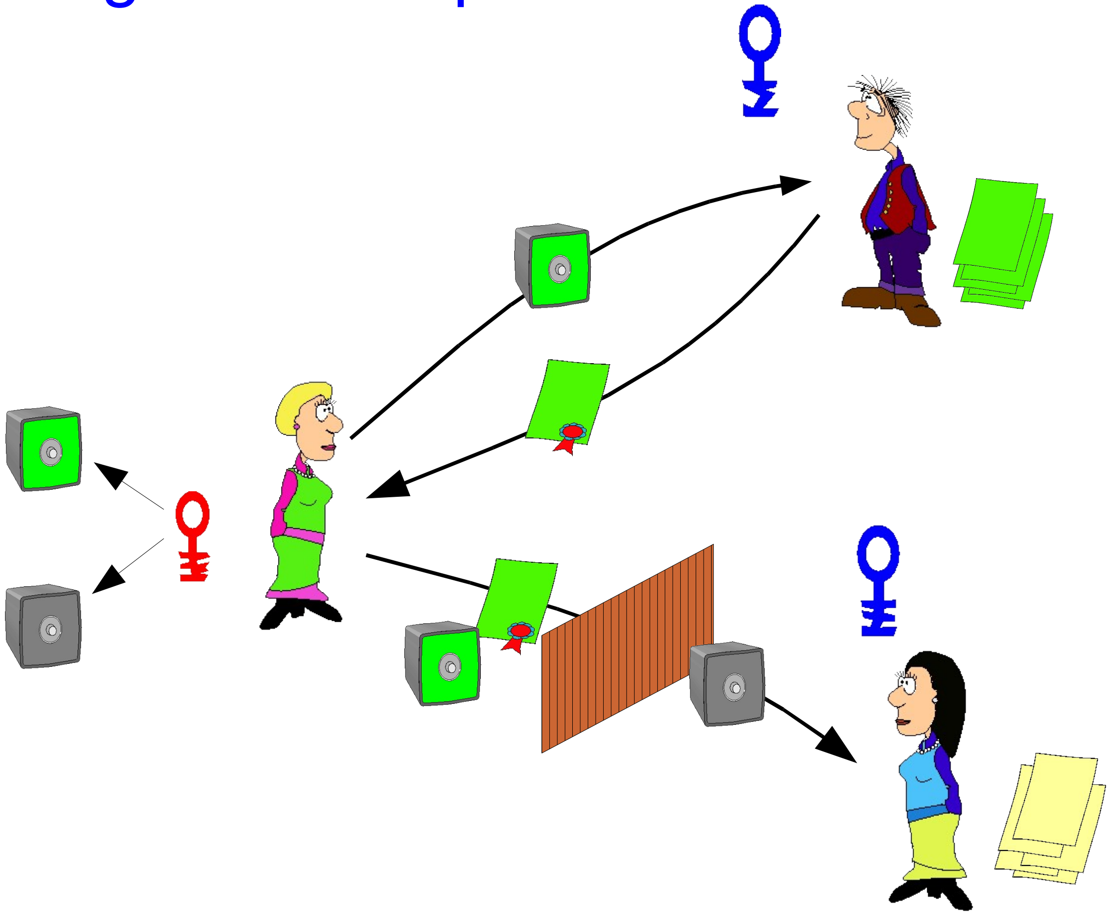

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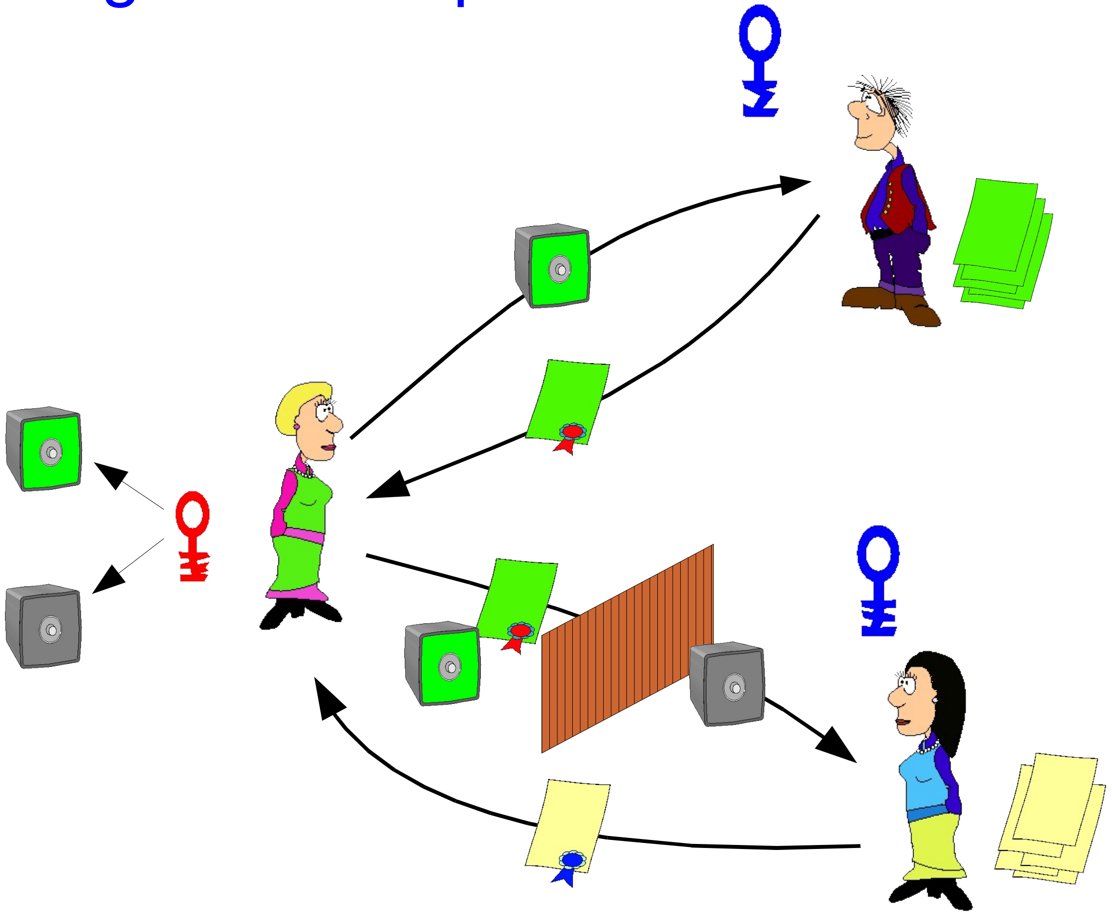
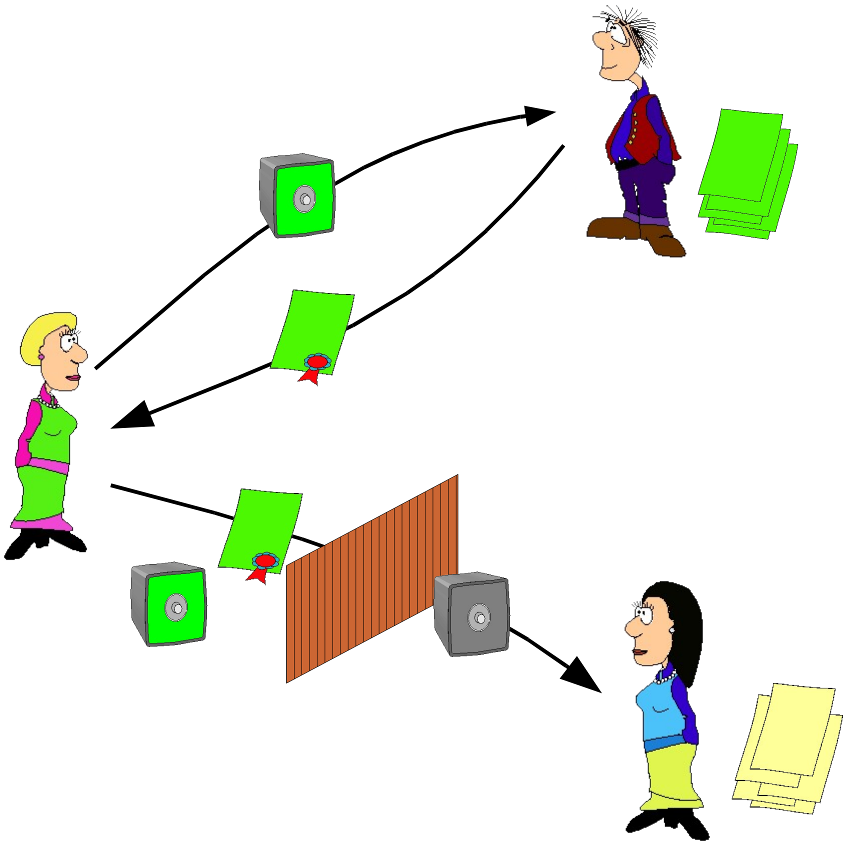
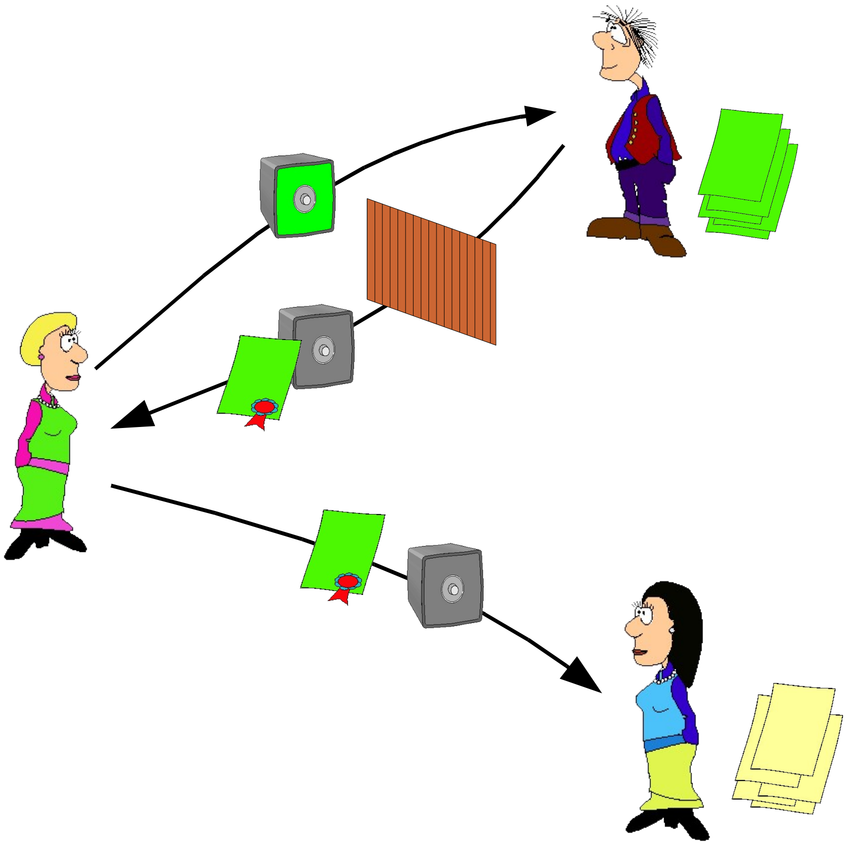


Proving Ownership Vs. Using Blind Signatures

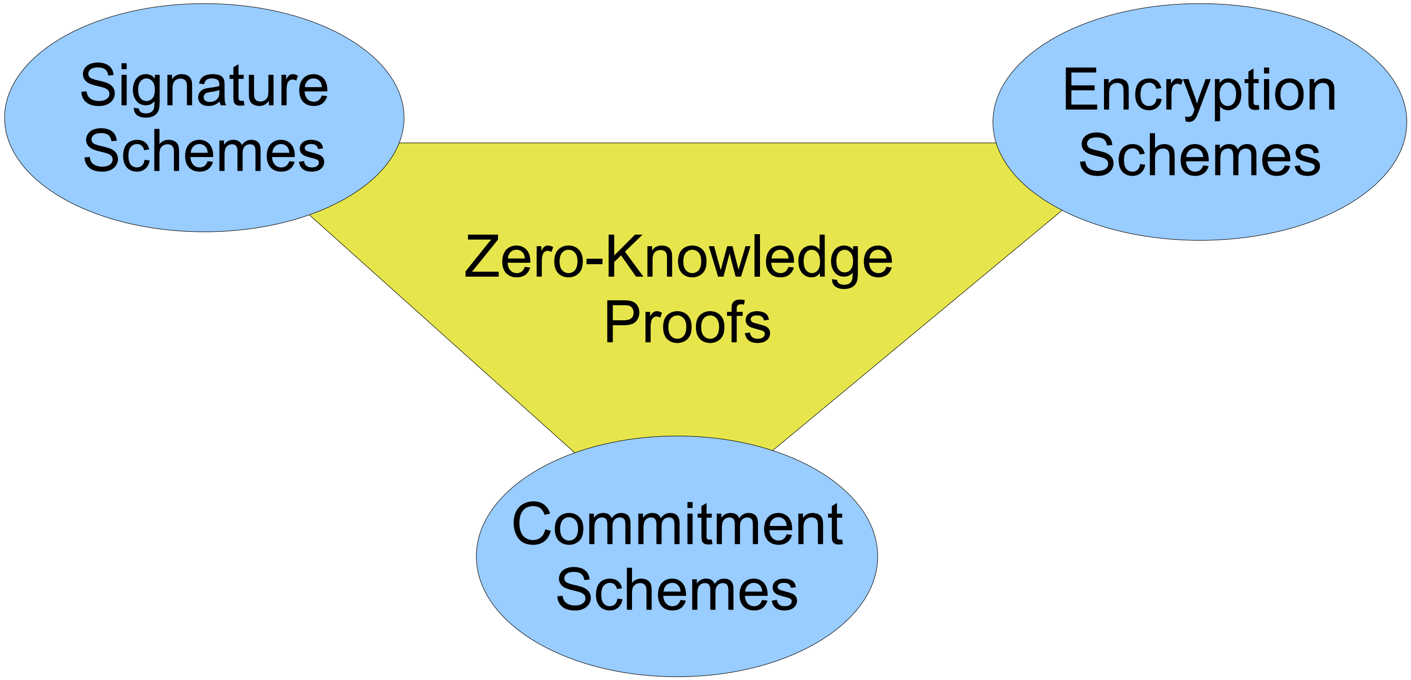




Certificates can be used *multiple* times!

Certificates can be used only *once*!

Required Technologies



..... challenge is to do all this efficiently!

Zero-Knowledge Proofs of Knowledge of Discrete Logarithms

[Schnorr '91,Chaum & Pedersen '92,....]

Given group $\langle g \rangle$ and element $y \in \langle g \rangle$.

Prove *knowledge* of $x = \log_g y$ such that verifier only learns y and g.

$PK\{(\alpha): \ y = g^{\alpha}\}$

Prover:		Verifier:
random r		
$t := g^r$	$\xrightarrow{\quad t \quad}$	
	$\xleftarrow{\quad c \quad}$	random c
$s := r - cx$	$\xrightarrow{\quad s \quad}$	
		$t = g^s y^c$

Zero Knowledge Proofs II

Non-interactive (Fiat-Shamir heuristic):

$$PK\{(\alpha): \; y = g^{\alpha}\}(m)$$

Logical combinations:

$$PK\{(\alpha,\beta): \; y = g^{\alpha} \wedge z = g^{\beta} \wedge u = g^{\beta}h^{\alpha}\}$$

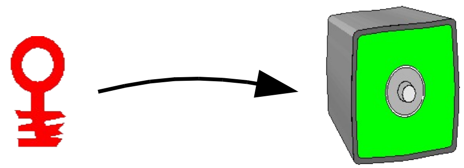
$$PK\{(\alpha,\beta): \; y = g^{\alpha} \vee z = g^{\beta}\}$$

Intervals and groups of different order (under SRSA):

$$PK\{(\alpha): \; y = g^{\alpha} \wedge \alpha \in [A,B]\}$$

$$PK\{(\alpha): \; y = g^{\alpha} \wedge \mathcal{z} = \mathcal{g}^{\alpha} \wedge \alpha \in [0,\min\{\mathrm{ord}(g),\mathrm{ord}(\mathcal{g})\}]\}$$

Commitment Schemes



Group $G = \langle g \rangle = \langle h \rangle$ of order q

To commit to element $x \in Z_q$:

- perfectly hiding, computationally binding (Pedersen):

  choose $r \in Z_q$ and compute $c = g^x h^r$

- computationally hiding, perfectly binding:

  choose $r \in Z_q$ and compute $c = (g^x h^r, g^r)$

To commit to integer $x \in Z$ (Damgård, Fujisaki):

- similarly, if order of G is not known, e.g., $G = QR_n$

The Strong RSA Assumption

Flexible RSA Problem: *Given RSA modulus* n *and* $z \in QR_n$ *find integers* e *and* u *such that*

$$u^e = z \bmod n$$

- Introduced by Barić & Pfitzmann '97 and Fujisaki & Okamoto '97
- Hard in generic algorithm model [Damgård & Koprowski '01]

Signature Scheme based on the SRSA Assumption I

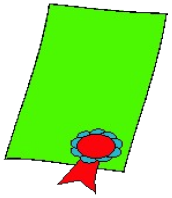
Public key of signer: RSA modulus n and a_i, b, $d \in QR_n$,

Secret key: factors of n

To sign k messages $m1, \ldots, mk \in \{0,1\}^{\ell}$:

- choose random prime $e > 2^{\ell}$ and integer $s \approx n$
- compute c such that

$$d = a_1^{m1} \cdot \ldots \cdot a_k^{mk} \, b^s \, c^e \mod n$$

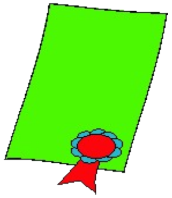


- signature is (c,e,s)

Signature Scheme based on the SRSA Assumption II

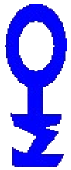
A signature (c,e,s) on messages $m1, \ldots, mk$ is valid iff:

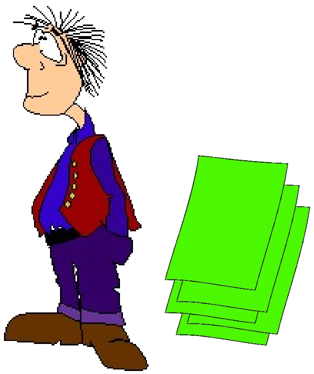
- $m1, \ldots, mk \in \{0,1\}^{\ell}$:
- $e > 2^{\ell}$
- $d = a_1^{m1} \cdot \ldots \cdot a_k^{mk} \; b^s \; c^e \bmod n$

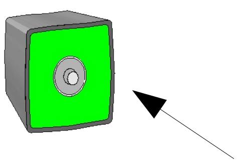


Theorem: *Signature scheme is secure against adaptively chosen message attacks under SRSA assumption.*

Getting a Signature on a Secret Message

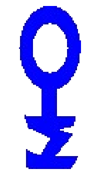


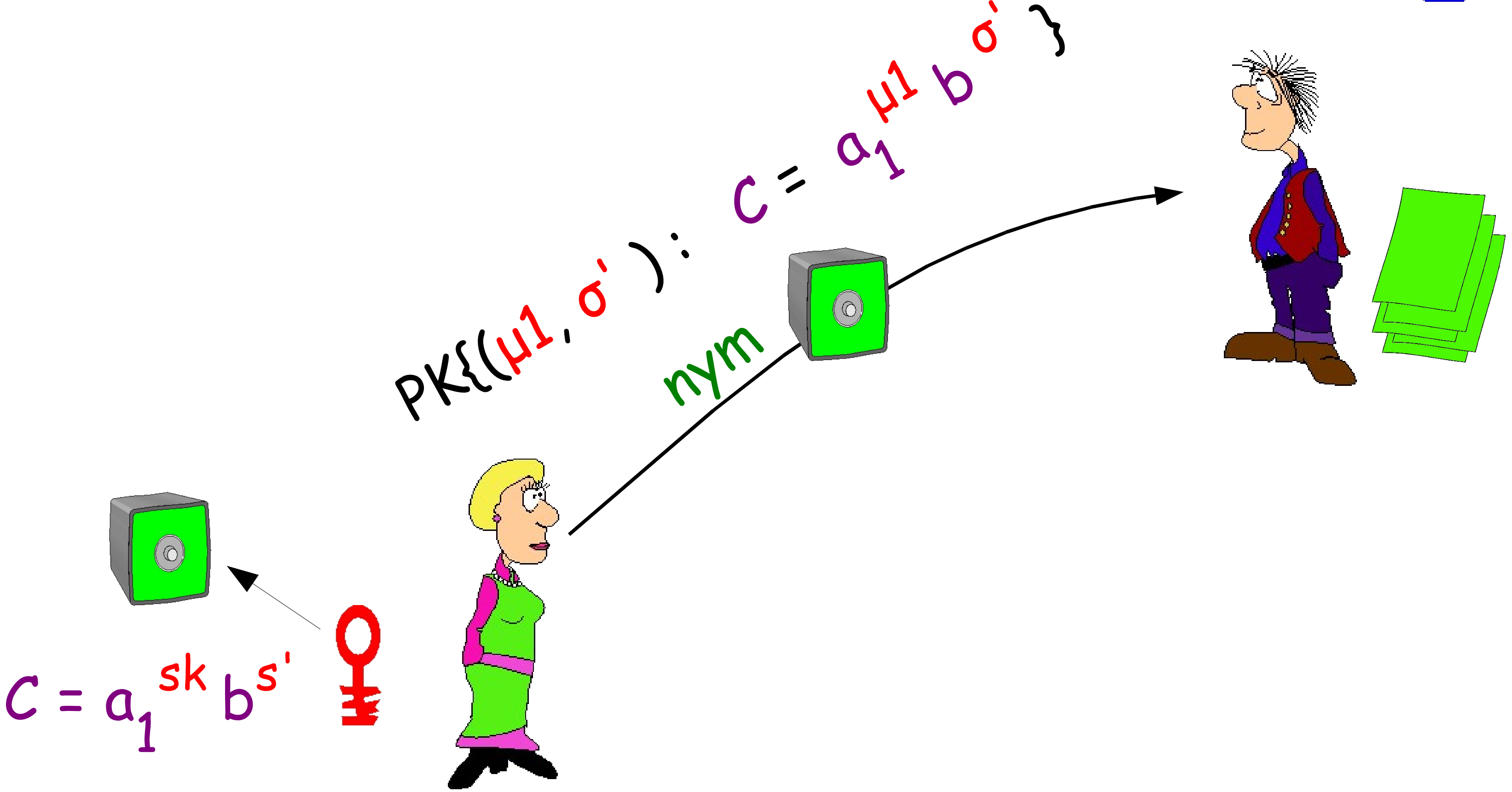




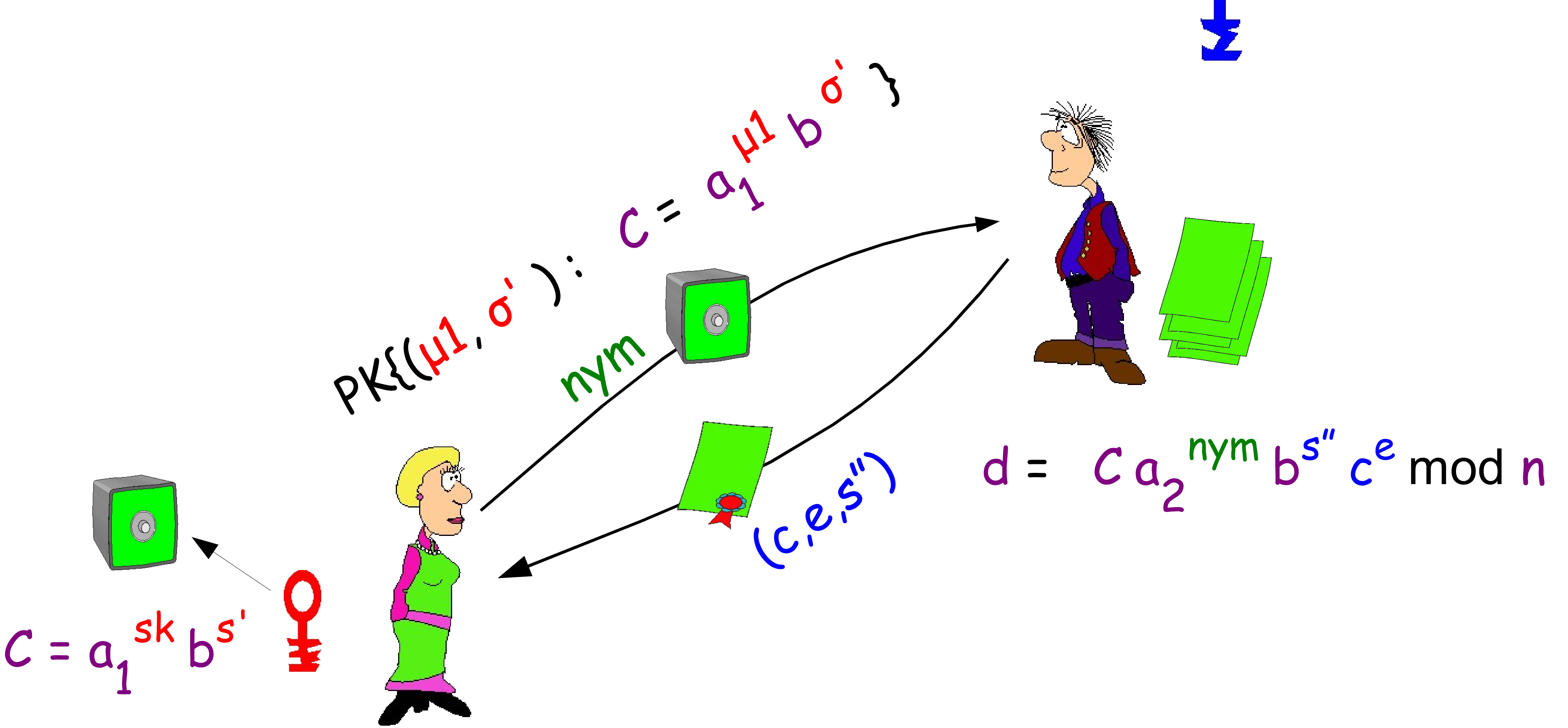
$C = a_1^{sk} b^{s'}$

Getting a Signature on a Secret Message



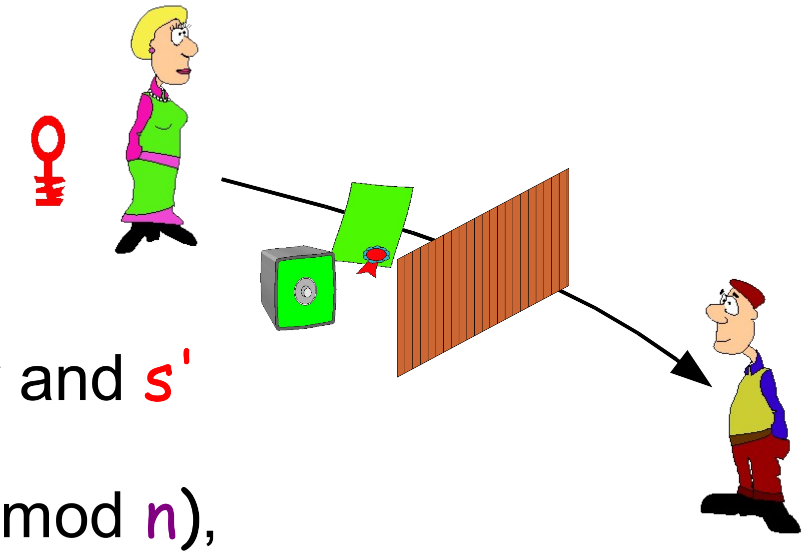


Getting a Signature on a Secret Message



$$d = a_1^{sk} a_2^{nym} b^{s'' + s'} c^e \pmod{n}$$

Proof of Knowledge of a Signature



Observe:

- Let $c' = c\, b^{s'}$ mod n with randomly and s'
- then $d = \; c'^{e} \; a_1^{m1} \cdot \ldots \cdot a_k^{mk} \; b^{s^*}$ (mod n),

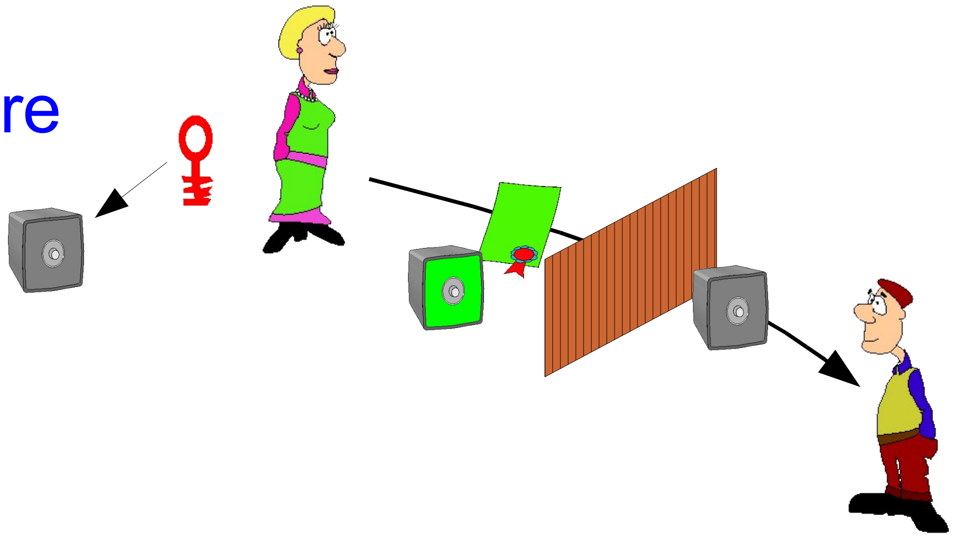
  i.e., (c', e, s^*) is a also a valid signature!

Therefore, to prove knowledge of signature on some m

- provide c'
- $PK\{(\varepsilon, \mu 1, \ldots, \mu k, \sigma) : \quad d := c'^{\varepsilon} a_1^{\mu 1} \cdot \ldots \cdot a_k^{\mu k} b^{\sigma}$

  $\wedge \quad \mu 1 \in \{0,1\}^{\ell} \quad \wedge \quad \varepsilon \in 2^{\ell+1} \pm \{0,1\}^{\ell} \quad \}$

Proof of Knowledge of a Signature



Using second Commitment

– $C = a_1^{sk} b^{s*}$

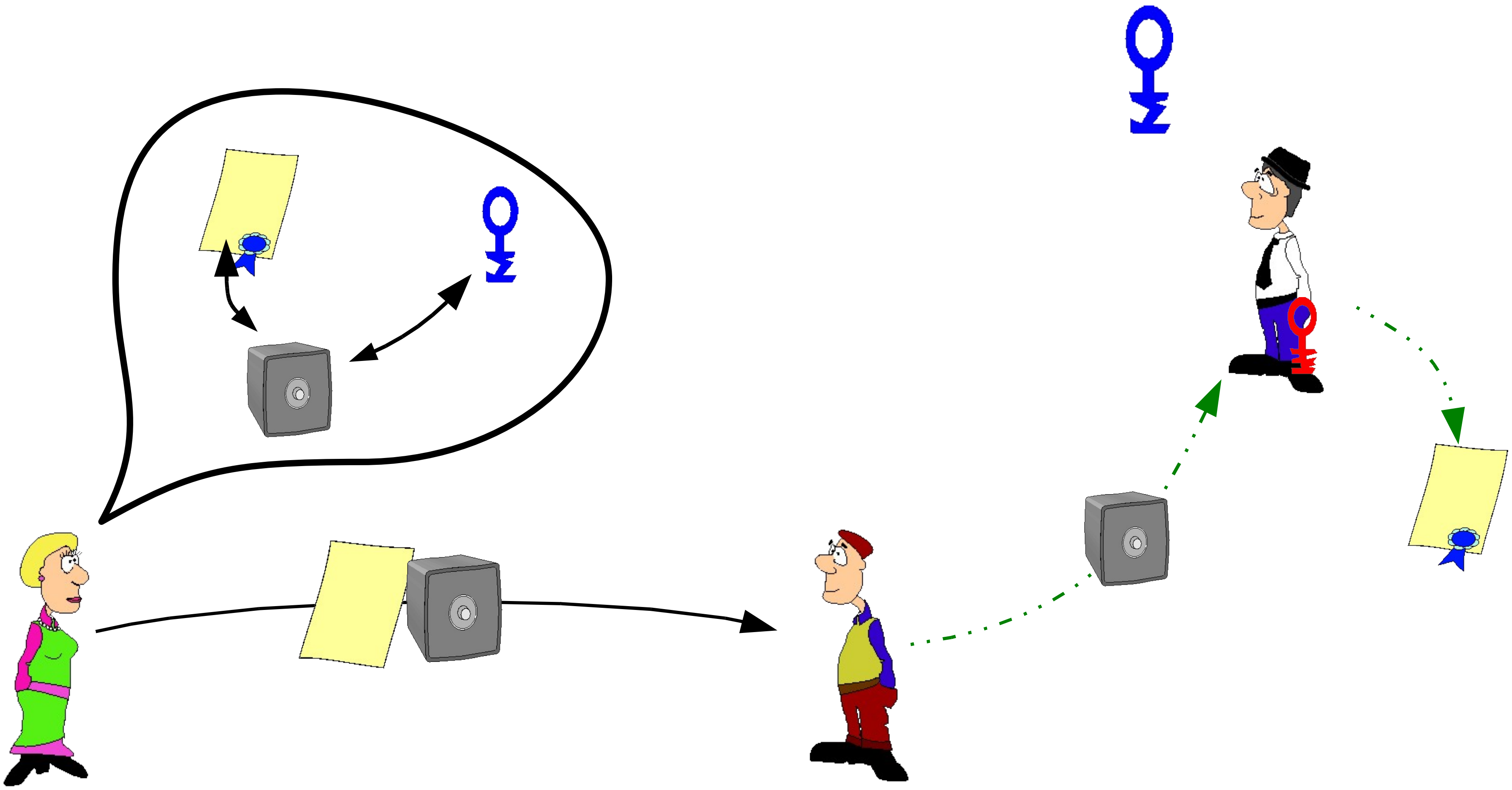
To prove knowledge of signature on some m

– provide c'

– PK{(ε, μ1, ..., μk, σ, σ*,) :

$$C = a_1^{\mu 1} b^{\sigma *} \quad \wedge \quad d := c'^{\varepsilon} a_1^{\mu 1} \cdot \ldots \cdot a_k^{\mu k} b^{\sigma} \}$$

Verifiable Encryption



The Decision Composite Residuosity Assumption

The DCR Problem: *Given n and x, decide whether or not*

$$x \in (Z^*_{n^2})^n \; .$$

- Introduced by Paillier '99.
- If $n = (2p'+1)(2q'+1)$ then $Z^*_{n^2} = Z_2 \times Z_2 \times Z_n \times Z_{p'q'}$.
- $(1+n)^u = (1+un) \bmod n^2$.

An Encryption Scheme

Public Key: n and $g, y_1, y_2, y_3 \in \langle (g')^{2n} \rangle$, where $g' \in Z^*_{n^2}$,

Secret Key: $x_i = \log y_i$

Encryption message $m \in [0,n]$ under label L:

- choose $r \in [0,n/4]$
- $u := g^r$, $e := y_1^r (1+n)^m$, $v :=$ abs$(y_2 y_3^{H(u,e,L)})^r$
- output (u,e,v).

where abs() maps ($a \bmod n^2$) to ($n^2 - a \bmod n^2$) if $a > n^2/2$, and ($a \bmod n^2$) otherwise, where $0 < a < n^2$.

An Encryption Scheme

Decryption of ciphertext (u,e,v) under label L:

- verify $v = \mathrm{abs}(v)$ and $u^{2(x_2 + H(u,e,L)x_3)} = v^2$.
- $\hat{c} := (e/u^{x_1})^{2t}$ where $t = 2^{-1} \bmod n$,
- if $n \mid (\hat{c}-1)$ output $m := (\hat{c}-1)/n$, otherwise output $\perp$

Intuition: remember $(1+n)^a = 1+an \pmod{n^2}$

so $(e/u^{x_1}) = y_1^r (1+n)^m / (g^r)^{x_1} = (1+n)^m = 1+mn$)

Theorem: *Encryption scheme is secure against adaptively chosen ciphertext attacks under DCR assumption.*

Verifiable Encryption of a Discrete Logarithm

Let $d = a_1^{sk} a_2^{nym}\ b^s\ c^e \pmod n$ be a driver's license

and (u,v,e) be an encryption of nym.

To prove that (u,v,e) indeed encrypts m:

$PK\{(\varepsilon, \mu 1, \mu 2, \rho, \sigma):$

$$d := c'^{\varepsilon}\ a_1^{\mu 1}\ a_2^{\mu 2}\ b^{\sigma} \ \wedge\ \mu 1, \mu 2 \in \{0,1\}^{\ell} \quad \wedge$$

$$u^2 = g^{2\rho} \wedge\ e^2 = y_1^{2\rho}\ (1+n)^{2\mu 2} \wedge\ v^2 = (y_2 y_3^{H(u,e,L)})^{2\rho} \quad \}$$

Conclusion & Outlook

- Efficient Anonymous Credentials and more!
- TCG TPM V1.2 will have some of this

  Was known in theory; soon your computer will have it.

- EU Project PRIME will have all of this

  www.prime-project.eu.org

- Plans:
  - Open source
  - Lots of more research :-)

Thanks for your attention!