Fortified Multi-Party Computation: Taking Advantage of Simple Secure Hardware Modules

Abstract: In practice, there are numerous settings where mutually distrusting parties need to perform distributed computations on their private inputs. For instance, participants in a first-price sealed-bid online auction do not want their bids to be disclosed. This problem can be addressed using secure multi-party computation (MPC), where parties can evaluate a publicly known function on their private inputs by executing a specific protocol that only reveals the correct output, but nothing else about the private inputs. Such distributed computations performed over the Internet are susceptible to remote hacks that may take place during the computation. As a consequence, sensitive data such as private bids may leak. All existing MPC protocols do not provide any protection against the consequences of such remote hacks. We present the first MPC protocols that protect the remotely hacked parties’ inputs and outputs from leaking. More specifically, unless the remote hack takes place before the party received its input or all parties are corrupted, a hacker is unable to learn the parties’ inputs and outputs, and is also unable to modify them. We achieve these strong (privacy) guarantees by utilizing the fact that in practice parties may not be susceptible to remote attacks at every point in time, but only while they are online, i.e. able to receive messages. To this end, we model communication via explicit channels. In particular, we introduce channels with an air-gap switch (disconnect-able by the party in control of the switch), and unidirectional data diodes. These channels and their isolation properties, together with very few, similarly simple and plausibly remotely unhackable hardware modules serve as the main ingredient for attaining such strong security guarantees. In order to formalize these strong guarantees, we propose the UC with Fortified Security (UC#) framework, a variant of the Universal Composability (UC) framework.

Keywords: universal composability, remotely unhackable hardware modules, security notions, isolation

1 Introduction

Secure multi-party computation (MPC), which allows mutually distrusting parties to securely evaluate a predefined function on their private inputs via a protocol, is a central privacy-enhancing technology that can be used in a wide range of scenarios. Examples include
- auctions (protecting the bids’ confidentiality) [8],
- contact discovery (protecting the contacts’ confidentiality) [35],
- toll collection (protecting e.g. location data) [21],
- medicine (protecting highly sensitive data such as (parts of) the genome) [3, 48],
- electronic payment (ensuring privacy and e.g. unlinkability) or [7],
- voting (e.g. confidentiality of the votes and correctness of the vote count) [46].

Protocol parties may become corrupted, i.e. fall under adversarial control. This can happen prior to the start of the protocol execution, which is called static corruptions. Furthermore, protocol parties may also become corrupted during the protocol execution via remote hacks. This type of corruption is called adaptive corruptions, first proposed in [13]. Up to now, all MPC constructions leak the private inputs of parties corrupted during protocol execution. However, the fact that the adversary learns all secrets of a corrupted party should be of a major concern, as there are plenty of sensitive MPC applications where this results in substantial real-world damage. In this work, we provide constructions where the adversary does not necessarily learn the inputs of parties corrupted via remote hacks during the protocol execution. Furthermore, this also holds for the corrupted parties’ outputs.

As an example, consider two government agencies from different countries that want to perform a private set intersection to learn who is on both countries’
We use the informal notion of an input port to denote the channel via which a party can receive its first input. In such a setting, it is very plausible that i) both parties can protect their devices against attacks requiring direct physical access but ii) are, at the same time, still susceptible to remote hacks from a nation-state attacker. Previous MPC protocols, even ones that are adaptively UC-secure, would not protect the remotely hacked party’s inputs’ and outputs’ privacy and integrity. Surprisingly, such strong and novel guarantees are possible and provided by our constructions.

We achieve this by exploiting the so-far overlooked observation that in all likely real-world scenarios, protocol parties who are not connected to the Internet just cannot be remotely “hacked”. For instance, a party may use data diodes (unidirectional channels) or disconnect itself via air-gap switches. With this, we can now introduce a distinction (motivated from the real world but with considerable theoretical interest) between remote hacks (e.g. sending malware), called online attacks in the following, and so-called physical attacks (e.g. replacing a part of the hardware or exploiting the larger attack surface provided by physical access to the device).

In conjunction with data diodes and air-gap switches, a protocol party may also use additional remotely unhackable hardware modules. An example of such an additional hardware module is a simple encryption unit that only implements a specific public key encryption scheme. All hardware modules used in our constructions are of very limited functionality and could be potentially formally verified for correctness, making it plausible to assume them to be resilient against remote hacking. In particular, an adversary can only corrupt such modules if he has direct physical access. Our assumptions are backed by commercially available devices with the required or similar functionality, see Section 5.

Utilizing only few and simple remotely unhackable hardware modules, we provide constructions with very strong security guarantees against online attacks. More specifically, online attacks i) mounted after a party received its first input and ii) mounted before a party received input if the attack comes from the “outside” do not allow an adversary to learn a corrupted party’s inputs and outputs nor to modify them, unless all parties are corrupted. Here, the “outside” denotes all channels except one at a party’s input port. In more detail, the parties in our protocols are disconnected from the outside while waiting for input and can therefore not be corrupted via online attacks from the outside at that point. After receiving input (via the input port), the parties authenticate, mask and share their secrets in such a way that mounting online attacks gives the adversary control over a party but not the ability to learn the party’s inputs or outputs, nor to modify them unless he gains control over all parties. This stands in contrast to adaptive corruptions where an adversary may learn and modify the inputs and outputs of corrupted parties after they received input. Although the possibility for parties to perform a secure erasure seems necessary for such a strong protection, we show that this assumption can be dropped in the full version. Also, the security of some of our constructions gracefully degrades to (essentially) standard adaptive UC security if the assumptions about the remotely unhackable hardware modules, channels and secure erasures turn out to be wrong (cf. Appendix A). This illustrates that security is not a binary property. We thus see our work as a first step towards a nuanced view of cryptographic security, which can be seen as a step towards the quantification of security.

In a bit more detail, we introduce a method for fortifying any generic MPC protocol\(^2\) secure in the presence of adaptive corruptions, so that the resulting protocol provides the above-mentioned strong (privacy) guarantees. The constructions we present provide these guarantees even when arbitrary (possibly malicious) protocols are executed concurrently.

In order to adequately capture the guarantees provided by our remotely unhackable hardware modules in a concurrent setting, we propose UC with Fortified Security, a variant of the UC framework [11]. Like UC security, the notion of UC with Fortified Security is based on the simulation paradigm where the execution of the protocol under analysis is compared to an ideal protocol execution. In the ideal protocol, all (honest) parties do not communicate with each other but only hand their private inputs to a trusted third party which locally evaluates the desired function and privately sends the results back to the parties. The protocol under analysis, called the real protocol, is said to be secure if it “emulates” the ideal protocol, i.e. any attack that can be mounted in the real protocol can also be carried out in the ideal protocol. Stated differently, for any given adversary against the real protocol, there exists an adversary against the ideal protocol, called the simulator, that can cause the same damage in the ideal protocol.

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1 We use the informal notion of an input port to denote the single channel via which a party can receive its first input.

2 Informally, a generic MPC protocol such as [14, 29] can be used to securely realize practically any efficiently computable function.
as the given adversary can cause in the real protocol. As a consequence, all security guarantees of the ideal world such as privacy carry over to an execution of the protocol under analysis. This way, there is no need to define a security notion for each desired security property, as they are captured by the specification of the ideal protocol.

For conciseness, we introduce the abbreviation UC# for “UC with Fortified Security”, where the added fence # symbolizes our “fortified” security. In this new variant of the UC framework, we deviate from the standard communication model and restrict the allowed communication in order to incorporate isolation properties.

A party's current online state (capturing whether the party can receive messages) is determined by the type and state of its channels, e.g., the (opened/closed) state of its air-gap switches. This will be made precise in Definition 1. The hardware modules used in a protocol and their connections are part of what we call the protocol architecture.

These changes of the communication model make it necessary to re-prove the composition theorem as well as other properties of the UC framework. We can show that our notion is equivalent to standard adaptive UC security if no remotely unhackable hardware units and isolation assumptions are used.

We stress that our remotely unhackable hardware modules should not be confused with the tamper-proof hardware tokens proposed in [30], as our hardware modules are a substantially weaker assumption. In particular, they can be tampered with if one has direct physical access. They cannot be passed to other, possibly malicious parties, but are only used and trusted by their owner. They are thus not sufficient and not intended to be used to circumvent the impossibility results of [12, 15]. In summary, our protocols provide the best possible protection against online attacks in a setting where parties cannot be protected while waiting for input.

1.1 Our Protocols in a Nutshell

Our protocols proceed mainly according to the following template, which consists of three phases. In the initial phase, starting for each party after it obtains input, the parties go offline (including their input port) and send encrypted and signed shares of their inputs via data diodes. In this phase, the encrypted shares are received by hackable buffers since the parties are offline and hence cannot receive messages themselves. Being offline, they also cannot retrieve the public keys necessary for encrypting the shares. This is therefore done by a remotely unhackable encryption unit (connected to its party via a data diode), which encrypts and sends the shares to the other parties’ buffers. At the end of this phase, each party erases each share except for its own and also erases the signing key used for authenticating their shares. Subsequently, a new phase begins where the parties are online and use the shares they received to obtain an encryption of the desired results. If an adversary corrupts a party via an online attack during this phase he only learns one share of each party’s input and only an encryption of that party’s output. Also, he is unable to modify the party’s input since the signing key has been erased. Finally, the desired result is decrypted and output in a final phase. This is done by modularizing each party into a hackable “core component”, which is referred to as the “party”, and a remotely unhackable output interface module (OIM) that verifies and decrypts the desired result. In more detail, the core component sends two symmetric keys, one for encryption and one for message authentication, to its OIM in the first phase. These two keys are shared along with the input. In the second phase, the core component receives the desired result (from the generic MPC protocol) encrypted and authenticated with these two keys. In the final phase, the core component hands the encrypted and authenticated result to its OIM, which verifies the authenticity of the encrypted result and if correct, decrypts and outputs the result. Since the OIM is remotely unhackable and the two keys stored in the OIM are shared among the parties, the adversary would have to corrupt every party, i.e. every core component, in order to be able to learn and modify a party’s output.

In order to highlight the necessity of remotely unhackable hardware modules and their isolation properties, we present two simple constructions in the following. Consider the execution of a secret-sharing-based MPC protocol such as [14] where the protocol parties \( P_1 \) and \( P_2 \) are initially honest. First, \( P_1 \) creates random shares \( s_1, s_2 \) of its input \( x_1 \), erases \( x_1 \) and sends \( s_2 \) to \( P_2 \) via a secure channel. At this point, the adversary remotely hacks \( P_1 \) and learns both \( s_1 \) and \( s_2 \) and is able to reconstruct the secret input \( x_1 \). This attack is not possible in our protocol as the machine holding \( s_1 \) is still offline at this point and not susceptible to remote hacks. This inability to protect the initially honest parties’ secret in case of remote hacks is intrinsic to previous MPC protocols.

Second, consider a protocol for secure two-party computations where \( P_1 \) and \( P_2 \) have access to a trusted execution environment (TEE). In the protocol of [43],
We introduce the so-far overlooked distinction between physical module. Both constructions can be proven secure which performs the actual computation and returns the der adversarial control (Theorems 2 and 4). For simplic-
also present an augmentation of these constructions that allow simulation also in the case that all parties are un-
allow in order to sends its input to the TEE, it is vulnerable at this point.

1.2 Our Contribution

We introduce the so-far overlooked distinction between physical and online attacks to protect the privacy-sensitive inputs and outputs of the participants in an MPC protocol against the latter, vastly more common type of attack. Furthermore, we utilize realistic simple remotely unhackable hardware modules that, to the best of our knowledge, have not been used for secure generic multi-party computation so far. With this, we also argue for a shift in how the security of the user's device is incorporated more concretely into secure computations.

Using only very few simple remotely un-hackable hardware modules, we construct MPC protocols that provide very strong guarantees against online attacks. More specifically, an adversary mounting online attacks i) after a party received its first input and ii) before a party received input if the attack comes from the “outside”, i.e. from all channels except one at a party’s input port, is unable to

- learn a party’s inputs and outputs
  (Strong Privacy)
- or to modify a party’s inputs and outputs
  (Strong Integrity)
unless he gains control over all parties.

We present a construction for non-reactive functionalities (Theorem 1) using only two simple remotely un-hackable hardware modules (apart from air-gap switches and data diodes) per party and a protocol for reactive functionalities (Theorem 3) that uses only one additional simple remotely un-hackable hardware module. Both constructions can be proven secure in our framework for adversaries that gain control over all but one party and feature graceful degradation. We also present an augmentation of these constructions that allow simulation also in the case that all parties are under adversarial control (Theorems 2 and 4). For simplic-
ity, we assume erasing parties. However, we show how this assumption can be dropped in the full version.

To properly formalize our above-mentioned guarantees, we propose a variant of the UC framework that adequately captures the advantages provided by remotely un-hackable hardware modules. In this endeavor, we have to take additional care to obtain a composable security notion, with some modeling choices that will be discussed in Section 2. Our security notion is equivalent to adaptive UC security for protocols that do not use any remotely un-hackable hardware modules. As a consequence, UC-secure protocols can be used as building blocks in our framework.

1.3 Related Work

Adaptive Security [13] captures security against adversaries that can corrupt parties at any point in the protocol. This notion has received considerable attention in the literature, see e.g. [14, 16, 27, 29]. In contrast to adaptive security where an adversary may learn all secrets of a corrupted party, we achieve that remotely hacking a party after it received its first input does not impact the confidentiality and integrity of a party’s inputs and outputs, unless all parties have been corrupted.

Mobile Adversaries [4, 39], a notion strictly stronger than adaptive security, models an adversary taking over participants (in a way similar in spirit to our “remote hacks/virus attacks”, although not modeling the online state) and possibly undoing the corruption later.

Concerning the used trusted building blocks, we assume data diodes, which are channels which allow for communication only in one specified direction. Garg et al. [23] analyze the cryptographic power of unidirectional channels as a building block, whereas we use unidirectional channels as a shield against dangerous incoming data packets. Achenbach et al. [1] make use of other trusted building blocks, such as a secure equality check module, to ensure the correct, UC-secure functioning of a parallel firewall setup in case of one malicious firewall.

Tamper-proof hardware tokens, first proposed by Katz [30], are an interesting research direction for finding plausible and minimal UC setup assumptions. Along this line of research, Goyal et al. [25] showed strong feasibility results of what can be done with these tokens. Moreover, Döttling et al. [18] showed that UC security is possible with a constant number of untrusted and resettable hardware tokens. Furthermore, [28] gives constructions of constant-round adaptively secure protocols which allow all parties to be corrupted. Most
tokens considered in the literature require token functionalities that are custom and do not capture readily available hardware (sign-and-commit in [30], bit one-time memory in [25], combined zero-knowledge and signature verification in [18] and PRF-from-commitment in [28]). Such tokens’ functionalities are, apart from input/output capability, of similar complexity when compared to our remotely un hackable hardware modules. However, the latter constitute a significantly weaker assumption, as they do not have to be physically tamper-proof and only need to be trusted by their owner. Also, none of the protocols aim to protect the parties’ secrets in case of corruption.

Isolation is a general principle in IT security, with lots of research on isolation through virtualization, see e.g. [36]. Isolation in this way can be seen as a software analog of a trusted, remotely un hackable encryption module. Moreover, there is a wealth of literature on data exfiltration/side channel attacks to air-gaps including attacks based on acoustic, electromagnetic and thermal covert channels, cf. [55], which are not relevant to our work, as they are for protecting against outgoing communication from malicious internal parties, while we use data diodes/air-gap switches for the purpose of not being hackable from the outside. As an example for isolation, the Qubes OS provides strict separation between application domains, allowing to use an isolated GNU Privacy Guard (GPG) environment safely [47].

Trusted Execution Environments (TEEs) such as Intel SGX promise the existence of an incorruptible secure enclave where arbitrary computations can be performed in a secure and isolated way, even if the host system is compromised. Moreover, parties can establish a secure channel with a TEE to provide inputs and receive outputs of computations. Using attestation, a cryptographic proof for the identity of code running inside a TEE can be obtained. TEEs have been used in conjunction with a global common reference string to achieve composable generic multi-party computation [43]. The resulting protocol is practically efficient, as it essentially only requires protocol parties to perform attestation, send their inputs encrypted to the TEE and receive the encrypted output. All (complex) computations are then performed inside the TEE without additional cryptographic overhead. However, if the TEE is insecure, all security is lost as inputs and outputs will be exposed to the adversary. This is in stark contrast to classic distributed MPC protocols that at least protect inputs and outputs of honest parties in case of corruptions. Today’s available implementations such as Intel SGX have very complex (closed-source) implementations and suffer from a number of vulnerabilities. In order to be considered remotely un hackable, the whole software stack exposed to the outside would have to be considered secure, in particular when handling possibly malicious messages. Due to the high complexity of current implementations, this may be unrealistic per se. In any case, it becomes less plausible with more complex code running inside the TEE, unless special precautions (such as formal verification or memory-safe languages) are taken.

Tinfoil Chat (TFC) is a peer-to-peer messaging system that uses multiple devices and data diodes to protect endpoints from remote hacks resp. their consequences (by using data diodes to block outgoing messages of machines that may have been remotely hacked) [40]. In contrast to previous frameworks such as the UC framework, which do e.g. not capture the isolation properties provided by data diodes, our framework is suitable to analyze the security of TFC. Compared to the novel security guarantees of our protocols, the security of TFC (for which there is no formal proof) is weaker as the integrity of the parties’ outputs is not protected.

1.4 Outline

We start by explaining the framework that is suitable to capture our guarantees in Section 2, albeit in a simplified way that hides some of the subtleties and technicalities that are quick to arise in the UC setting, but are formally necessary to achieve composability of the security notion. Section 3 then states our main construction for the case of non-reactive functionalities, whereas the case of reactive functionalities is given in Section 4. For a discussion on graceful degradation, we refer to Appendix A. For the security proof of the presented construction, see Appendix B. In our exposition, we assume familiarity with the UC framework, but include a brief introduction to the UC framework in Appendix C.

2 Universal Composability with Fortified Security (UC#)

The advantages provided by un hackable hardware modules, e.g. the isolation properties of data diodes and air-gap switches, have not been considered in the context of secure multi-party computation before. The resulting challenge is twofold: First, the framework has to be able to express such properties adequately. Secondly, the security notion must be meaningful and exhibit proper-
ties such as universal composability. At the same time, both framework and security notion should be compatible with existing UC-secure protocols, allowing to re-use them without having to re-prove their security in the new framework.

Here, we give an overview of how we capture the advantages provided by remotely unhackable hardware modules, by explicitly modeling the connection topology between parties using standard channels, data diodes and air-gap switches, and their corresponding “online status”. Finally, we introduce necessary trusted interface modules that handle a (honest) party’s output, and describe our new security guarantees formally.

### 2.1 Network Topology, Channel Types and Online Status

In our model, we make use of the following channels:

**Standard Channels** between two entities\(^3\) allow ordinary bi-directional communication. They are depicted by a straight line connecting them:

\[
\begin{array}{c}
A \\
\hline
B
\end{array}
\]

We say that \(A\) and \(B\) are connected (via a standard channel).

**Data Diodes** between two entities allow for communication in one direction only. For entities \(A\) and \(B\), we depict this with a connection containing a diode sign in the direction of the permissible data flow, here from \(A\) to \(B\):

\[
\begin{array}{c}
A \\
\hline \\
\rightarrow
B
\end{array}
\]

We say that \(A\) is connected to \(B\) via a data diode (but not vice versa).

**Air-gap Switches** are channels between two entities that can be connected or disconnected once per activation by the entity that is in control of the switch. For entities \(A\) and \(B\), where \(A\) is in control of the switch, we depict this with a switch sign, where the operating entity is next to the hinge:

\[
\begin{array}{c}
A \\
\hline
B
\end{array}
\]

On the left the switch is disconnected, while on the right it is connected.Disconnected air-gap switches allow no data transmissions at all, while connected ones allow bi-directional communication. In both cases, we say that \(A\) is connected to \(B\) via an air-gap switch (but not vice versa; and we may add that it is connected or disconnected).

We allow for the possibility to have multiple channels of different type between two entities. We say that \(A\) can send messages or provide input/output to \(B\) (via \(C\)) if there is a channel \(C\) between \(A\) and \(B\) that allows the communication from \(A\) to \(B\). Note that if there is no channel between two entities, then no communication is allowed between them. Channels can be between (sub-)parties of a protocol or between a (sub-)party and an ideal functionality. In addition, channels can also be between a party and the environment or the adversary. Channels between a party and the environment model the allowed communication with calling parties (from other protocols). Channels between a party and the adversary model possible communication to the “outside world” that can be “delivered” by the adversary.

As in the UC framework, the adversary \(A\) and the environment \(Z\) may freely interact with each other. The same applies to the communication between \(A\) and ideal functionalities. Formally, we always assume standard channels between these instances of interactive Turing machines (ITIs). Communication between these ITIs is therefore independent of the given protocol architecture.

**Online State of the Parties.** We now define the online state of the parties via the possibility of receiving messages or input/output:

**Definition 1** (Online State). A (sub-)party \(P\) of protocol \(\pi\) is online (via \(C\)) if there is a channel \(C\) such that one of the following holds:

1. \(P\) can receive messages from the adversary via \(C\).
2. \(P\) can receive output from a functionality \(F\) via \(C\).
3. \(P\) can receive in- or output via \(C\) from a sub-party or calling party \(M\), resp., and \(M\) is online via another channel \(C'\) from \(M\) and a party distinct from \(P\).
4. \(P\) can receive input from the environment \(Z\) via \(C\).

If there is no such channel \(C\), we say that \(P\) is offline. Here, item 1 models that \(P\) can receive messages from the “outside world”. Item 2 captures that messages can be received from a trusted third party \(F\) that “lives” somewhere in the outside world, such as a public bulletin board or a common reference string. Item 3 models a party being transitively online via connections to other parties who are online. Item 4 models a party being transitively online via connections to other parties who are online.
transitively online via connections to a calling party (i.e. a party from another protocol which provides input).

Each time the adversary \( A \) is activated, he gets informed via which channels each party is online. This is called the status. As will be described in Section 2.3, \( A \) will be able to gain control over (hackable) parties when they are online.

### 2.2 The Protocol Architecture

The protocol architecture of a protocol \( \pi \) specifies the set of all channels involving the parties of \( \pi \), together with the initial connection state of each air-gap switch, and for each (sub-)party in \( \pi \) whether it is (remotely) hackable or unhackable. Formally, the architecture of \( \pi \) is part of \( \pi \)'s code.

In our figures of protocol architectures, main parties are represented by boxes with rounded corners, sub-parties by cornered ones, ideal functionalities are enclosed in a cloud, and the adversary is enclosed in a circle. Boxes with a double border denote that the corresponding (sub-)party is unhackable. Channels that end at the bottom are channels to the environment.\(^4\)

**Example 1** (Online State). An example of a protocol architecture is given in Fig. 1 (left). There, the protocol starts with two main parties \( P_1, P_2 \), where \( P_1 \) and \( P_2 \) are initially online (\( P_1 \) and \( P_2 \) due to item 4 and \( P_2 \) also due to items 1 and 3), as well as online subparties \( Q_1, Q_2 \) (online due to item 2, \( Q_2 \) also due to item 3). We use the same figures to also depict later protocol states. Typically, a party \( P_1 \) will disconnect its air-gap switch to the environment \( Z \) as soon as it has received input (cf. Fig. 1 middle), making it offline. Later, \( P_1 \) may connect its air-gap switch to the adversary \( A \) at a specific point, say, after having erased its input (cf. Fig. 1 right), making it online again.

### 2.3 Corruption Model

As motivated above, we make use of the fact that in reality there is a significant difference between remote hacks (e.g. by an exploit that is executed upon a computer parsing some input message, such as an email with attached malware), and the much rarer and more difficult to perform physical attacks, where a computer’s hardware is tampered with, or the adversary has physical access to the device. This allows us to provide stronger protection in the first case, whereas previous corruption models over-pessimistically assume the latter, hence resorting to less remaining privacy and security guarantees in cases of a corruption. This fits to our plausible assumption that some simple hardware modules can be implemented such that they cannot be remotely hacked. Thus, in our model, an adversary \( A \) is given the option to perform

- **Physical Attacks**, where the targeted main party, and all its subparties, fall under adversarial control, as usual in adaptive security, and
- **Online Attacks**, where only the targeted (sub-)party falls under adversarial control, but only if it is online and not assumed to be (remotely) unhackable.

If \( A \) has gained control over a (sub-)party \( P \) through one of these attacks, we say that \( P \) is corrupted. In this case, we (pessimistically) assume that \( A \) has access to \( P \) even if \( P \) disconnects its air-gap switches, by formally creating a new standard channel between \( P \) and \( A \).

In order to keep the proofs and discussions simple, we will restrict ourselves to a simplified adversarial model where physical attacks are only allowed prior to the start of the protocol. This is motivated by the fact that physical attacks (i.e. tampering with hardware) are time consuming and therefore typically must be mounted before the start of the protocol execution. However, our constructions for up to \( N - 1 \) corrupted parties remain secure even if physical attacks are allowed throughout the execution, which we will only cover in the full version to keep the exposition simple.

In the following, we describe our new corruption model in more detail.\(^5\) Let \( \mathcal{P} \) be the set of main parties of a protocol \( \pi \). At the first activation, the adversary \( A \) may only send a physical-attack instruction that enables him to gain control over parties regardless of the protocol architecture. Formally, \( A \) writes (physical-attack, \( \mathcal{M} \)), where \( \mathcal{M} \subseteq \mathcal{P} \), on his outgoing message tape. Each \( P \in \mathcal{M} \) and all of its sub-parties

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\(^4\) These figures capture interactive Turing machines (ITMs) only. When an ITM instance (ITI) is invoked, run-time information such as the session ID is accounted for when determining the channels between ITIs of the same protocol, the environment (resp. the calling protocol) and the adversary.

\(^5\) Note that the following describes the behavior of protocol parties in the real model upon receiving corruption messages. As in the UC framework, in ideal protocols the behavior upon party corruption is determined by the ideal functionality.
are then connected to the adversary via a standard channel and all air-gap switches controlled by and data diodes coming from these parties are replaced with standard channels. From then on, \( \mathcal{A} \) has full control over all \( P \in \mathcal{M} \) and all of their sub-parties.

From the second activation on, the adversary may not send a physical-attack instruction anymore. \( \mathcal{A} \) may send online-attack instructions that enable \( \mathcal{A} \) to gain control over hackable parties when they are online. Formally, if \( \mathcal{A} \) writes \((\text{online-attack}, P)\) on his outgoing message tape and \( P \) is a (sub-)party of \( \pi \) that is online and hackable, then a standard channel between \( P \) and \( \mathcal{A} \) is created and all air-gap switches controlled by \( P \) are connected. \( P \) then sends its entire local state to \( \mathcal{A} \). From then on, \( \mathcal{A} \) has full control over \( P \). If \( P \) is unhackable, then this instruction is ignored.

Finally, if a (sub-)party \( P \) is corrupted, then each ideal functionality \( \mathcal{F} \) which is connected to \( P \) is informed about \( P \) being corrupted through a special message \((\text{corrupt}, P)\) that is written on \( \mathcal{F} \)'s incoming message tape. Also, each main party immediately informs the environment after being corrupted.

### 2.4 Interface Modules

In order to achieve the strong security guarantees mentioned previously, a party’s result of the MPC must remain unmodified and hidden from the adversary \( \mathcal{A} \) even if the party is corrupted via an online attack after receiving input. This is not possible if a party learns its result and outputs it itself since \( \mathcal{A} \) would learn this result if he corrupts the party and could then also instruct the party to output a modified value. Furthermore, for reactive tasks, a party corrupted after receiving input (via an online attack) must also not be able to learn or modify its input(s) for the rounds \( \geq 2 \).

Deviating from the UC framework, we therefore allow the main parties to invoke special sub-parties called interface modules that are connected to their main party as well as to the environment via channels specified by the protocol architecture. These interface modules may thus give subroutine output to or receive input from the environment subject to the protocol architecture. Intuitively, interface modules model simple hardware modules connected to, e.g., a PC. During the protocol execution, a user does not trust his PC since it may have been remotely hacked (in particular, the output of his PC may have been altered by a hacker). Instead, he only trusts the unhackable interface modules and, in particular, the outputs given by them, e.g., via a display.

In our constructions, interface modules will be unhackable sub-parties with very limited functionality. We will assume an interface module called output interface module (OIM) that is used for ensuring that a party’s result of the MPC remains unmodified and hidden from the adversary even in the case that the party is corrupted via an online attack after receiving input. More specifically, a party’s result will only be learned by its OIM, which outputs the result instead of the party.

For reactive tasks, we will also assume an input interface module (IIM) that is used for ensuring that a party’s input for the rounds \( \geq 2 \) remain secret and unmodified even in the case that the party is corrupted after receiving (its first) input. In the ideal execution, the ideal functionality may also interact with dummy parties corresponding to interface modules (Definition 2).

### 2.5 Our Formal Security Notion

In this section, we give an ideal-model protocol architecture tailored to our security guarantees and define fortified functionalities. Recall that, as described in
Section 2.3, $\mathcal{F}$ is informed through a special message (corrupt, $P_i$), which is written on its incoming message tape, when a party $P$ connected to $\mathcal{F}$ is corrupted. We now define the protocol architecture for our ideal protocol $\mathcal{A}(\mathcal{F})$, which stands for “Air-Gap switches” that are used there. It is defined differently dependent on whether $\mathcal{F}$ is non-reactive or reactive, i.e. whether $\mathcal{F}$ interacts with the parties in a single round, taking at most one input from each party and providing at most one output to each party, or whether it may receive inputs and provide outputs in multiple rounds (possibly keeping state between rounds), respectively.

For a non-reactive functionality $\mathcal{F}$, $\mathcal{A}(\mathcal{F})$ is the ideal protocol where $N$ hackable “dummy main parties” $P_1, \ldots, P_N$ are connected to $\mathcal{F}$ via an initially disconnected air-gap switch and to the environment via an initially connected air-gap switch and additionally $N$ unhackable “dummy output interface modules” OIM$_1, \ldots, $OIM$_N$ which are connected to $\mathcal{F}$ and the environment via standard channels, see also Fig. 2 (left).

Upon input $v_i$, each party $P_i$ disconnects its air-gap switch to $Z$, connects its air-gap switch to $\mathcal{F}$, and passes $v_i$ to $\mathcal{F}$. Each $P_i$ connects its air-gap switch to $Z$ again upon receiving a special message connect from $\mathcal{F}$. Furthermore, if $\mathcal{F}$ is reactive, $\mathcal{A}(\mathcal{F})$ additionally contains $N$ unhackable “dummy input interface modules” IIM$_1, \ldots, $IIM$_N$ which are connected to $\mathcal{F}$ via initially disconnected air-gap switches controlled by $\mathcal{F}$ and to the environment via initially disconnected air-gap switches, see Fig. 2 (right) for reference. Each IIM$_i$ connects its air-gap switch to the environment upon receiving connect from $\mathcal{F}$, after $\mathcal{F}$ has connected its air-gap switch to IIM$_i$.

As the air-gap switch between a party $P_i$ and $\mathcal{F}$ is disconnected before $P_i$ has received input, the parties $P_i$ in $\mathcal{A}(\mathcal{F})$ cannot be corrupted by an online attack “coming from the outside” (i.e., through channels except the input port) prior to receiving input. More specifically, each $P_i$ can only be corrupted by an online attack prior to receiving input if it is online via its channel to the environment. In the following we will also refer to OIM$_i$ (and IIM$_i$) as the “dummy OIM (resp. IIM) of $P_i$". Moreover, we call an ideal functionality $\mathcal{F}$ standard if $\mathcal{F}$ i) immediately notifies the adversary upon receiving input from an (honest) party, and ii) is standard corruption$^6$, and iii) only gives delayed outputs to parties.

2.5.1 Fortified Functionalities

In contrast to functionalities in the adaptive UC security model, fortified functionalities do not pass the inputs and outputs of a party $P_i$ corrupted after receiving input to the adversary $A$ and also do not allow him to modify $P_i$’s input or the output to $P_i$’s dummy OIM, unless all parties $P_j$ ($j = 1, \ldots, N$) are corrupted. $A$ can only block an output or instruct the functionality to pass either the computed output or an error symbol $\perp$ to $P_i$’s dummy OIM. If all parties are corrupted, $A$ learns all inputs and outputs and may modify them arbitrarily (including the outputs to the dummy OIMs). This is formally captured in the following definition, where we omit session IDs for simplicity.

Definition 2 (Fortified Functionality). Let $\mathcal{G}$ be a non-reactive standard ideal functionality interacting with $N$ parties $P_1, \ldots, P_N$ and $\mathcal{A}$. Define the fortified functionality $[\mathcal{G}]$ of $\mathcal{G}$ interacting with $A$, $P_1, \ldots, P_N$, and their dummy OIMs OIM$_1, \ldots, $OIM$_N$ as follows:

- $[\mathcal{G}]$ internally runs an instance of $\mathcal{G}$.
- $[\mathcal{G}]$ initializes a counter $c = 0$.
- Upon receiving input from $P_i$, $[\mathcal{G}]$ forwards it to $\mathcal{G}$.
- Each time $\mathcal{G}$ sends a notification to $A$ upon receiving input from an (honest) party, $[\mathcal{G}]$ forwards that notification to $A$.
- $[\mathcal{G}]$ forwards all delayed outputs of $\mathcal{G}$ for $P_i$ to $A$.
- Upon confirmation by $\mathcal{A}$, $[\mathcal{G}]$ forwards the output to OIM$_i$.
- Upon receiving (corrupt,$P_i$), $[\mathcal{G}]$ increments $c$, marks $P_i$ as corrupted before input if $[\mathcal{G}]$ has not

---

$^6$ It proceeds as follows upon receiving a (corrupt,$P$) message from $A$. First, $F$ marks $P$ as corrupted and outputs corrupted to $P$. In the next activation, $F$ sends to $A$ all the inputs and outputs of $P$ so far. In addition, from this point on, whenever $F$ gets an input value $v$ from $P$, it forwards $v$ to $A$, who may then send a “modified input value” $v'$ that overwrites $v$. Also, all output values intended for $P$ are sent to $A$ instead.
yet received input from \( P_i \), or as corrupted after input, otherwise, and forwards (corrupt, \( P_i \)) to \( G \). The subsequent output (corrupted) for \( P_i \) from \( G \) is forwarded to \( P_i \).

- Handling parties marked as corrupted before input:
  - If \( G \) sends the input of \( P_i \) to \( A \), \([G]\) forwards that input to \( A \). Furthermore, if \( A \) sends a modified input value for \( P_i \), \([G]\) forwards that value to \( G \).
  - If \( G \) sends an output intended for \( P_i \) to \( A \), \([G]\) sends that output to \( A \). \( A \) may instruct \([G]\) to pass any output of his choice to OIM.

- Handling parties marked as corrupted after input:
  - If \( c < N \) and \( G \) sends the input of \( P_i \) to \( A \) after receiving (corrupt, \( P_i \)), \([G]\) ignores this message. Furthermore, if \( A \) sends a modified input value for \( P_i \), ignore this value.
  - If \( c < N \) and \( G \) sends the output intended for \( P_i \) to \( A \), \([G]\) first notifies \( A \) that OIM is about to receive output. \( A \) may then instruct \([G]\) to pass that output or \( \perp \) to OIM.

- Upon reaching \( c = N \), send all inputs and outputs to \( A \). \( A \) may then determine the outputs of all OIMs.
- All other messages between \( A \) and \( G \) are forwarded.
- If \( A \) sends (output, \( \tilde{y}, P_i \)), \([G]\) outputs \( \tilde{y} \) to \( P_i \) if \([G]\) has marked \( P_i \) as corrupted. (While the adversary cannot modify the output of the dummy OIM of a party corrupted after receiving input, unless \( c = N \), he is able to determine what a corrupted party itself outputs).

Fortified functionalities of reactive functionalities are defined in the full version. By construction, \( \Lambda(G([G])) \) captures our desired security goal: i) \([G]\) ensures that corrupting a party \( P_i \) via an online attack after it has received its input does not enable the adversary to learn or modify \( P_i \)'s input(s) and result(s) of the MPC (i.e. outputs of \( P_i \)'s dummy OIM), unless all \( N \) parties are corrupted, and ii) the initially disconnected air-gap switches between the parties \( P_i \) and \([G]\) ensure that the adversary cannot corrupt a party \( P_i \) via an online attack “coming from the outside” before \( P_i \) has received input, i.e. each \( P_i \) can only be corrupted via an online attack at that point if it is online via its channel to the environment.

### 2.5.2 Achieving a Meaningful Notion

In the UC framework, the adversary is not activated when a party provides input or receives subroutine output from a sub-party and is therefore not able to corrupt it during this communication. In our setting, this is undesirable as it does not capture the possibility of parties being hacked when they are online during this immediate communication. This can yield obviously insecure protocols that would be secure in our framework. To see this, consider a party \( P \) connected to \( Z \) and \( A \) as well as an unhackable sub-party \( P' \) via standard channels:

\[
\begin{array}{c}
P' \\
\downarrow \\
\downarrow \\
\downarrow \\
\downarrow \\
\downarrow \\
\downarrow \\
A \\
\end{array}
\]

Upon receiving input, \( P \) sends secret data to \( P' \). \( P' \) then sends a notification message to \( P \) who immediately erases all secret data after being activated again. As this message delivery is immediate, i.e. \( A \) is not activated during this communication, he is unable to corrupt \( P \) before \( P \) has erased its secret data even though \( P \) has been online the entire time. To address this problem, we introduce a notify transport mechanism in the full framework that activates the adversary (under certain conditions) upon immediate message delivery.

We can now define security in UC# in analogy to the UC framework:

**Definition 3 (Emulation in the UC# Framework).** Let \( \pi, \phi \) be protocols. \( \pi \) emulates \( \phi \) in the UC# framework, denoted by \( \pi \geq_{\text{UC#}} \phi \), if for every PPT adversary \( A \) there is a PPT adversary \( S \) such that for every PPT environment \( Z \) there is a negligible function \( \text{negl} \) such that for all \( n \in \mathbb{N}, a \in \{0, 1\}^* \) it holds that

\[
\Pr[\text{Exec}_{\text{UC#}}(\pi, A, Z)(n, a) = 1] - \Pr[\text{Exec}_{\text{UC#}}(\phi, S, Z)(n, a) = 1] \leq \text{negl}(n),
\]

where \( \text{Exec}_{\text{UC#}}(\pi, A, Z)(n, a) \) denotes the random variable for the environment \( Z \)'s output in the UC# execution experiment with protocol \( \pi \) and adversary \( A \) on input \( a \) and security parameter \( n \).

Let \( \pi \) be a protocol with main parties \( P_1, \ldots, P_N \). Then, \( \pi \) emulates \( \phi \) for up to \( L \leq N \) parties under adversarial control if emulation holds for all (real-model) PPT adversaries \( A \) corrupting \( L \) main parties of \( P_1, \ldots, P_N \).

### 2.6 Properties of the Framework

As in the UC framework, the dummy adversary is complete in UC#. Our security notion is also transitive. UC# security is equivalent to adaptive UC security for "plain" UC protocols that are appropriately embedded
into UC#, i.e., their security also holds in UC#. For the full properties and proofs, we refer to the full version.

Achieving Composability. Note that for universal composability, we have to introduce a few additional twists to the framework that allow us to prove the composition theorem. Let us elaborate. In the UC framework, the environment models all protocols that run concurrently with the protocol under analysis (challenge protocol). Naturally, the parties in the protocols incorporated by the environment also have online states that may change dynamically during the execution of these protocols. The online states of the parties incorporated by the environment may influence the online states of the parties in the challenge protocol. Therefore, the environment must be able to modify the online states of its channels to the challenge protocol depending on the online states of the parties it incorporates.

Consequently, we allow the environment to additionally set its channels to the parties to activated or deactivated. We therefore change item 4 in Definition 1 by stipulating that a party is online via a channel $C$ if it can receive input from the environment via $C$ and $C$ has been set to “activated” by the environment. Additionally, we provide the environment, for each of its channels, the information whether it can currently receive output from that channel. With these changes, our security notion can be shown to be closed under protocol composition just like UC security. This is given in the full version.

3 Construction for Non-Reactive Functionalities

In this section, we present the construction $\Pi_G$ for realizing the fortified functionality $[G]$ of any non-reactive, standard, and adaptively well-formed\(^7\) functionality $G$. The broad idea is to have the parties $P_1, \ldots, P_N$ send encrypted shares of their inputs via data diodes in an offline sharing phase and subsequently use these shares to compute the desired function in an online compute phase. This, however, cannot be done straightforwardly.

To begin with, the parties are not able to retrieve public keys themselves in the sharing phase since this would necessitate going online, making them susceptible to online attacks. Therefore, each party $P_i$ sends its shares to an unhackable sub-party called encryption unit (Enc unit) via a data diode. The Enc unit retrieves the public keys and sends encrypted shares to hackable sub-parties of the designated receivers, called buffers.

Furthermore, each message has to be authenticated so that the adversary cannot change the input of a party. One could do this with an additional unhackable “authentication unit” which signs each ciphertext or have the Enc unit sign all ciphertexts. However, since we want to use as few and as simple unhackable sub-parties as possible, we take a different approach. Each party $P_i$ sends its shares together with valid signatures to its Enc unit. The verification key is sent, over an intermediary sub-party called join (J), to a hackable sub-party called registration module (RM) that disconnects itself from J after receiving input and forwards the verification key to a public bulletin board ($\mathcal{F}_{\text{reg}}$) via a data diode. Once a party $P_i$ has sent all of its shares, it erases everything except for its own share, its verification key and its decryption key. In order for this sign-then-encrypt approach to be secure, we assume that the PKE scheme is non-malleable (IND-parallel-CCA-secure)\(^6\) and that the digital signature is unforgeable (EUF-naCMA secure) and also satisfies a property we call length-normality, guaranteeing that signatures of messages of equal length are also of equal length. The latter property prevents an adversary from learning information of plaintexts based on the length of their ciphertexts. Each party $P_i$ is connected to its sub-party $J$ via an initially disconnected air-gap switch in order to prevent the adversary from corrupting $P_i$’s RM but not $P_i$ before $P_i$ has received its input.

In the compute phase, the adversary must be prevented from using values that are different from the shares sent by the honest parties to the corrupted parties in the sharing phase. Otherwise, he would be able to modify the inputs of the parties who were honest during the sharing phase. The parties $P_i$ therefore not only use the shares they received but also the signatures of these shares and the registered verification keys during the compute phase. The result of the compute phase is a special “error symbol” if not all signatures are valid. Since the signing keys were erased at the end of the sharing phase, the adversary cannot generate new valid signatures for parties $P_i$ corrupted after receiving input. He is also unable to revoke the verification key of such

\(^6\) An ideal functionality is adaptively well-formed if it consists of a “shell” and a “core”. The core is an arbitrary PPT TM. The shell is a TM that acts as a “wrapper”: All incoming messages are forwarded to the core except for corrupt messages. Furthermore, outputs generated by the core are forwarded by the shell. Additionally, the shell sends the random tape of the core to the adversary if all parties are corrupted at some activation.
parties since this would require corrupting the respective RM, which is impossible since that party is offline. Moreover, an adversary could swap a message in the sharing phase addressed to (the buffer of) an honest party \( P_i \) with a ciphertext of a share and signature received by a corrupted party (by encrypting that tuple with the respective public key). Furthermore, an adversary controlling at least two parties \( P_i, P_j \) knows two shares and valid signatures of each party and could use one of these tuples twice in the compute phase. To prevent these attacks, a party \( P_i \) signs each share along with the designated receiver’s PID, which will be denoted by “\( P_j \)” in the following. In addition, a party \( P_i \) also includes its own PID in each message it sends to prevent the adversary from reusing messages sent by honest parties for parties corrupted before receiving input.

Finally, one cannot simply send the result of the compute phase to a party \( P_i \) since this would allow the adversary to learn and modify the output of the parties corrupted after receiving input. Instead, we use an un hackable output interface module (OIM). Each party \( P_i \) sends not only the shares of its input \( x_i \) but also shares of a random pad \( r_i \) and of a MAC key \( k_i \) in the sharing phase. Furthermore, each party \( P_i \) sends \( r_i \) and \( k_i \) to its OIM via a data diode. In the compute phase, the parties will then use these shares to compute \((y_i + r_i, \text{Mac}(k_i, y_i + r_i))\), where \( y_i \) is the resulting output value of (party \( P_i \)) and + denotes the bit-wise XOR. Each party then sends its result to its OIM, which will check authenticity by verifying the MAC tag and, if correct, reconstruct and output the value \( y_i \).

In the following, we will take a modular approach (as facilitated by the composition theorem of UC\#) and define – for an ideal functionality \( G \) representing an MPC – an ideal functionality \( \mathcal{F}_G \) that implements the verification of the input values in the compute phase as well as the subsequent multi-party computation (of \( G \)) on the shares. Applying the UC\# composition theorem, we are able to replace \( \mathcal{F}_G \) with an existing adaptively UC-secure protocol, e.g. [14]. Note that this will require an additional setup assumption, e.g. a common reference string, as our un hackable sub-parties, channels, and \( \mathcal{F}_{\text{reg}} \) are not UC\#-complete.

Functionality 1 (\( \mathcal{F}_G \)). Let \( G \) be a non-reactive standard adaptively well-formed ideal functionality. \( \mathcal{F}_G \) proceeds as follows, running with parties \( P_1, \ldots, P_N \) and adversary \( A \) and parametrized with a digital signature scheme \( \text{SIG} \) and a message authentication code \( \text{MAC} \).

1. Initialize the Boolean variable \( \text{verify} = \text{true} \).
2. Upon receiving input from \( P_i \), store it, mark \( P_i \) as input given and send (received, \( P_i \)) to \( A \). Ignore further input of \( P_i \).

Consistency Check:

3. Once all parties are marked as input given, check if each stored input is of form \( \overline{vk}_i = (vk_i^{(1)}, \ldots, vk_i^{(N)}) \), \((s_j, r_j, k_j, \sigma_j) (j = 1, \ldots, N) \). If not, set \( \text{verify} = \text{false} \). Otherwise, check if \( \overline{vk}_i = \cdots = \overline{vk}_N \).
   (a) If the check fails, set \( \text{verify} = \text{false} \).
   (b) Else, set \( (vk_1, \ldots, vk_N) = (vk_1^{(1)}, \ldots, vk_1^{(N)}) \).
   Check if \( \text{Vrfy} \Sigma(G)(vk_j, (P_i, s_j, r_j, k_j), \sigma_j) = 1 \) for all \( i, j \in \{1, \ldots, N\} \).
   i. If the check fails, set \( \text{verify} = \text{false} \).
   ii. Else, proceed with item 4.

Reconstruction and Computation:

4. For \( i = 1, \ldots, N \), compute \( x_i = s_{i1} + s_{i2} + \cdots + s_{iN} \), \( k_i = k_{i1} + k_{i2} + \cdots + k_{iN} \) and \( r_i = r_{i1} + r_{i2} + \cdots + r_{iN} \).
5. Internally run \( G \) on input \((x_1, \ldots, x_N)\). Let \((y_1, \ldots, y_N)\) be the output of \( G \). For all \( i = 1, \ldots, N \), compute \( a_i = y_i + r_i \) and \( \theta_i \leftarrow \text{Mac}(k_i, a_i) \).
6. If party \( P_i \) requests an output, proceed as follows:
   (i) If \( \text{verify} = \text{false} \), send a private delayed output \( \perp \) to \( P_i \).
   (ii) Else, if item 5 has already been carried out, send a private delayed output \((a_i, \theta_i)\) to \( P_i \).
7. \( \mathcal{F}_G \) is standard corruption (cf. Footnote 6). Once all parties are corrupted, \( \mathcal{F}_G \) sends its private randomness to \( A \). (This ensures that \( \mathcal{F}_G \) is also adaptively well-formed).
8. All other messages between \( A \) and \( G \) are ignored.

Let \( \mathcal{F}_{\text{reg}} \) be the public bulletin board functionality (cf. Appendix C.1 for a definition). Let \( \text{PKE} = (\text{GenPKE}, \text{Enc}, \text{Dec}) \) be a public-key encryption scheme, \( \text{SIG} = (\text{GenSIG}, \text{Sig}, \text{VrfySIG}) \) a digital signature scheme and \( \text{MAC} = (\text{GenMAC}, \text{Mac}, \text{VrfyMAC}) \) a message authentication code. Given a non-reactive standard adaptively well-formed functionality \( G \), we next define our protocol \( \Pi_G \) for realizing \( AG([G]) \).

Construction 1. Define the protocol \( \Pi_G \) as follows:

Architecture: See Fig. 3 for a graphical depiction. Figs. 4 and 5 show the changes of \( \Pi_G \’s \) state throughout the execution.

Offline Sharing Phase

Upon input \( x_i \), each party \( P_i (i = 1, \ldots, N) \) does:
- Disconnect the air-gap switch to the environment.
- Generate \((pk_i, sk_i) \leftarrow \text{GenPKE}(1^n), k_i \leftarrow \text{GenMAC}(1^n), (sgk_i, vk_i) \leftarrow \text{GenSIG}(1^n) \) and a random pad \( r_i \leftarrow \{0,1\}^{p_i(n)} \), where \( p_i(n) \) is an unspeci-
All air-gap switches, except for $P_i$'s air-gap switch to the environment (making it offline) and $P_i$'s air-gap switch to the RM's air-gap switch to the environment (making it offline) and $P_i$'s air-gap switch to the environment (making it offline) and $P_i$'s air-gap switch to the environment (making it offline), are connected to the adversary via standard channels. Enc unit $Enc_i$: Receive a list $L = \{(P_j, v_j)\}_{j=1}^{\{1, \ldots, N\} \setminus \{i\}}$ from one’s main party $P_i$. At each activation, for each $(P_j, v_j) \in L$, request $pk_j$ of $P_j$ from $F_{reg}$. If retrievable, compute $c_{ij} \leftarrow Enc(pk_j, v_j)$, send $(P_i, c_{ij})$ to buffer of $P_j$ and delete $(P_j, v_j)$ from $L$. Then, go into idle mode.

Buffer: Store each message received. On input retrieve, send all stored messages to one’s main party.

**Online Compute Phase**

After the sharing phase, a party $P_i$ does the following:

- Connect air-gap switches to $Buff_i$, $F_{reg}$ and $F_{G}$.
- Request all verification keys $\{vk_j\}_{j=1}^{\{1, \ldots, N\} \setminus \{i\}}$ from $F_{reg}$ registered by the other parties’ RMs. If not all verification keys can be retrieved yet, go into idle mode and request again at the next activation.
- Send retrieve to $Buff_i$ and check if it sends at least $N-1$ messages. If not, go into idle mode and, when activated again, send retrieve and check again. If yes, check if one has received from each party $P_j$ a set $M_j = \{(P_j, c)\}$ with the following property $(\ast)$ (Validity Check):

There exists a tuple $(P_j, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$ and a $(P_j, c) \in M_j$ such that:

- $Dec(sk_i, c) = (P_j, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$ and $Vrfy_{SIG}(vk_j, (P_i, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}), \hat{\sigma}_{ji}) = 1$
- For all $(P_j, \hat{c}) \in M_j$ either $Dec(sk_i, \hat{c}) = (P_j, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$, or $(P_j, \hat{c})$ is "invalid", i.e., either decrypts to a tuple $(P_j, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$ such that $Vrfy_{SIG}(vk_j, (P_i, \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}), \hat{\sigma}_{ji}) = 0$, or decrypts to a tuple $(P', \hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$ such that $P' \neq P_j$ or does not decrypt correctly.

If $(\ast)$ does not hold, send $\bot$ to $F_{G}$. Else, send all retrieved verification keys $(vk_1, \ldots, vk_N)$ as well as all tuples $(\hat{s}_{ji}, \hat{r}_{ji}, \hat{k}_{ji}, \hat{\sigma}_{ji})$ $(j \in \{1, \ldots, N\} \setminus \{i\})$ as well as $(s_{ii}, r_{ii}, k_{ii}, \sigma_{ii})$ to $F_{G}$.

**Online Output Phase**

Having completed its last step in the compute phase, a party $P_i$ requests output from $F_{G}$ and forwards that output to $OIM_i$.

---

8 Sending the sender’s PID as prefix is not necessary but simplifies the discussion. Note that for $(P_i, c)$ we also say that “$c$ is addressed as coming from party $P_i$.”
Fig. 5. $\Pi_\mathcal{G}$ during and after the online compute phase: First, $RM_i$ has disconnected its air-gap switch to $J_i$, keeping it offline. Then, $P_i$ has connected its air-gap switches to $F\mathcal{G}$, $F\text{reg}$ and $Buffer_i$.

$OIM_i$: Store the first input $(k_i, r_i)$ from one’s main party. On second input $(o_i, \theta_i)$ or $\perp$ from one’s main party, do the following: If the received value equals $\perp$, output $\perp$. Otherwise, check if $\text{Vrfy}_{\text{MAC}}(k_i, o_i, \theta_i) = 1$ and output $y_i = o_i + r_i$ if this holds, and $\perp$ otherwise.

**Remark 1.** Note that we do not model how to reuse modules such as $RM_i$ that stay disconnected throughout the protocol execution. In practice, one may assume, e.g., a physical reset mechanism for these modules.

**Theorem 1** (Up to $N − 1$ Corruptions, Non-Reactive). Let $\mathcal{G}$ be a non-reactive standard adaptively well-formed functionality. Assume $\text{PKE}$ is IND-pCCA-secure and $\text{SIG}$ is EUF-naCMA-secure and length-normal, and $\text{MAC}$ is EUF-1-CMA-secure. Then, for up to $N − 1$ parties under adversarial control, it holds that

$$\Pi_\mathcal{G} \geq \text{UC}_\# \text{AG}([\mathcal{G}]).$$

**Proof.** The proof will be given in Appendix B.

3.1 Up to $N$ Corrupted Parties

One can augment Construction 1 to obtain a protocol $\Pi_\mathcal{G}^N$ that is also secure if the adversary corrupts all parties at the expense of one additional unhackable sub-party called decryption unit (Dec unit). Note that all other constructions given in this paper are summarized in Fig. 6. The main idea in the new construction is that parties do not decrypt ciphertexts themselves but send them to their Dec unit. Each Dec unit receives the secret key from its main party during the sharing phase.

In the compute phase, each Dec unit accepts a single vector of ciphertexts from its main party. Since the Dec units are unhackable and do not leak the secret keys, the simulator can report plaintext tuples to $Z$ in such a way that the shares they contain are consistent with the parties’ inputs and outputs even if all parties are corrupted.

Fig. 6. Architecture for a) non-reactive functionalities and up to $N − 1$ parties under adversarial control, if red IIM/blue Dec unit parts are absent, b) non-reactive functionalities and up to $N$ parties under adversarial control, if the blue Dec unit part is absent, c) reactive functionalities and up to $N − 1$ parties under adversarial control, if the red IIM part is absent, and d) reactive functionalities and up to $N$ parties under adversarial control, if the red IIM/blue Dec unit parts are present.

**Theorem 2** (Up to $N$ Corruptions, Non-Reactive). Let $\mathcal{G}$ be a non-reactive standard adaptively well-formed functionality. Assume $\text{PKE}$, $\text{SIG}$, $\text{MAC}$ are as in Theorem 1. Then, for up to $N$ parties under adversarial control, it holds that

$$\Pi_\mathcal{G}^N \geq \text{UC}_\# \text{AG}([\mathcal{G}]).$$

4 Construction for Reactive Functionalities

For reactive functionalities, a new problem arises because parties are online after the first round. The input(s) for the next round(s) can therefore not just be given to a party since it may have been corrupted. We therefore need to find a way to insert the input(s) for round $u \geq 2$ into the protocol without allowing a party to learn or modify them.
To this end, we introduce an additional unhackable hardware module called \emph{input interface module} (IIM) that acts as the counterpart of the OIM for inputs. Inputs for round(s) \( n \geq 2 \) have to be inserted into the protocol via the IIM which masks each input it receives and computes a MAC tag of the padded input. In the compute phase, these MAC tags are verified along with the signatures of the shares (of random pads for the inputs/outputs and of a MAC key) (cf. the full version for details). Denote by \( \Pi_{\text{reac}}^N \) this new protocol.

**Theorem 3** (Up to \( N-1 \) Corruptions, Reactive). Let \( \mathcal{G} \) be a reactive standard adaptively well-formed functionality. Let PKE and SIG be as in Theorem 1 and assume that MAC is EUF-CMA-secure. Then, for up to \( N-1 \) parties under adversarial control it holds that
\[
\Pi_{\text{reac}}^N \geq_{\text{UC}} \mathcal{AG}(\mathcal{G}^\text{reac}).
\]

**4.1 Up to \( N \) Corrupted Parties**

With the same augmentation as described in Section 3.1, one can obtain a protocol \( \Pi_{\text{reac}}^{N,\text{reac}} \) that is also secure if the adversary corrupts all parties.

**Theorem 4** (Up to \( N \) Corruptions, Reactive). Let \( \mathcal{G} \) be a reactive standard adaptively well-formed functionality. Let PKE, SIG, MAC be as in Theorem 3. Then, for up to \( N \) parties under adversarial control, we have
\[
\Pi_{\text{reac}}^{N,\text{reac}} \geq_{\text{UC}} \mathcal{AG}(\mathcal{G}^\text{reac}).
\]

**5 Implementations of Remotely Unhackable Hardware Modules**

In contrast to hardware that can be used as UC setup, e.g., tamper-proof hardware tokens or TEEs, our remotely unhackable hardware modules constitute a much weaker assumption as i) they only have to be trusted by their owner, ii) do not need to be sent to other parties and iii) only have to be secure against remote hacks, meaning that they do not have to be tamper-proof.

In order to protect from remotely exploitable vulnerabilities, we suggest to use formal verification wherever possible. There exists a vast body in the literature that is applicable to the implementations we discuss below, like smart cards ([2, 5]), cryptographic implementations in the IoT world [45], FPGAs and ASICs [9, 20] or microkernels [26, 31]. Due to the very low complexity of the cryptographic core, formal verification is applicable in practice for our modules.

**Virtualization and TEEs.** As a first attempt, we consider the case where some or all modules are implemented on the same machine in software. To this end, there are two main approaches. The first considers a hypervisor [26, 31] that implements each component in a dedicated virtual machine. Assuming that i) the hypervisor is secure and provides isolation between the individual VMs and the host and ii) the components are implemented correctly, the resulting system can be considered a secure implementation of our construction. When not using dedicated hardware, security can be strengthened by using a TEE such as Intel SGX to implement some of the components, in particular the Dec unit, the OIM and the IIM in case of reactive computations.

**Data Diodes and Air-Gap Switches.** In principle, data diodes and air-gap switches can be implemented in software using packet-filtering firewalls. However, this approach is susceptible to vulnerabilities in the packet filter and the associated software stack. There also exist numerous commercial off-the-shelf hardware solutions for data diodes (e.g. [22, 24, 44]) and air-gap switches [32] that can be controlled remotely. However, some are rather complex and not easily verifiable for correctness. In contrast, there exist numerous open-source DIY implementations (e.g. [40, 52]) for data diodes that exploit physical principles and thus require very little trust. The data diode proposed by [40] can be easily adapted to different communication technologies.

A remotely controlled air-gap switch could also be achieved by using a micro-controller to control a relay which (dis)connects one or several wires of an Ethernet connection (cf. [41]) or, e.g., a RS-232 connection. Alternatively, one could also use any manageable Ethernet switch or a non-managed switch combined with a remotely controllable power strip.

**Enc and Dec Unit.** In contrast to data diodes and air-gap switches where only correctness and no privacy is required, the Enc unit additionally has to keep the share it receives secret. Due to the use of public-key encryption schemes only, the Enc unit does not have to handle secret keys. Additional complexity is introduced by having to retrieve public keys from a public key infrastructure (PKI), for which a network interface is required. As long as the PKI’s answers can be verified, e.g. by using certificates, the interface can be untrusted.
Due to its low complexity, the Enc unit can be implemented using off-the-shelf hardware with low complexity such as microcontrollers or single-board computers. While we are not aware of such hardware with formally verified firmware, we believe that the risk originating from firmware vulnerabilities is acceptable in this case—especially if the programming interface is not exposed. At the OS layer, implementations in memory-safe language such as Tock [33] in Rust are available. Such a memory-safe language could also be used for the actual protocol implementation. As current TEEs such as Intel SGX communicate via the host OS, we do not consider them appropriate for the Enc unit, whose purpose is to keep the host system offline at the onset of the computation. Instead, it would be susceptible to remote hacks, contradicting the purpose of the Enc unit.

For the Dec unit, being very similar to off-the-shelf hardware like hardware security module (HSM) (e.g. [54]) or a smart card that can store private keys and perform decryptions, we believe that HSMs and smart cards are very natural candidates. This additionally assumes an appropriate interface that also enforces that only a single ciphertext vector is decrypted. There also exist a number of open-source solutions that can be adapted. An example is the NetHSM by Nitrokey [38], whose whole software is written in the memory-safe language OCaml. As OS, MirageOS with the formally verified microkernel Muen [26] is used. The security token Solo 2 [50] has an extensible open-source firmware, Trussed [51], written in Rust and can similarly be adjusted.

The Dec unit could also be implemented using a field-programmable gate array (FPGA). In our setting, the FPGA does not have to be tamper-proof, as our goal is to protect from the consequences of remote hacks without physical access to the device. This also holds for side-channels, unless they can be exploited via the connection to the FPGA. The code on the FPGA could be designed using a special-purpose software like Cryptol [20], which allows the generation of VHDL code which provably adheres to its specification. Also, the security of the FPGA firmware has to be considered. Recent attacks such as [19] have shown that caution is necessary. Of course, the user also has to trust that its code is correctly deployed and executed by the FPGA. A number of FPGA-based HSMs with open-source firmware are available [17, 42, 49], which could also serve as basis.

It is also conceivable to implement the Dec unit using a TEE. However, this approach has the drawback that current implementations are very complex, relying on a combination of software, firmware and hardware that have to be trusted by the user due to their closed-source nature and may all contain potential vulnerabilities or even backdoors. In particular, side-channel attacks [37] that allow the extraction of secrets from a TEE may be a major problem in our setting, assuming that the adversary is able to compromise the host system where the Dec unit is running.

Common to complex building blocks such as TEEs or FPGAs is the difficulty or even inability of the user to verify that the correct code is actually executed.

OIM and IIM. Remember that OIM and IIM accept one or several one-time pads and MAC keys and are responsible for the verification and decryption of results (OIM) resp. encrypting and authenticating inputs (IIM). As OIM and IIM only provide interfaces for input resp. output, no network capability is required. Thus, the host connection can be realized using some simple protocol such as RS-232 that can be safely implemented.

We can also extend an HSM, security token or FPGA with the required functionality and connect it via an appropriate interface that provides secure I/O, e.g. using a simple dot-matrix display and a keyboard. By augmenting TEEs with secure I/O, e.g. [34, 53] for Intel SGX, OIM and IIM can be in principle realized using a TEE. However, we believe that these approaches for TEEs with secure I/O should be considered proof-of-concepts not yet fit for practice.

6 Conclusion

When using MPC in practice, one has to protect well against the consequences of remote hacks. Utilizing only few and simple remotely unhackable hardware modules and their accompanying isolation assumptions, we constructed general MPC protocols providing very strong composable security guarantees against online attacks.

Some of our protocols even exhibit graceful degradation, which is a step towards more nuanced, quantifiable security.

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References


2_19.
Appendix

A Graceful Degradation

In the following, we discuss the security implications of our constructions for the case of PID-wise corruption, i.e. the assumption that the protocol $\Pi^*$ provides the output of parties via their respective dummy module, it is not a legal protocol in the standard UC framework. For the same reason, we cannot hope to use the protocol of Construction 1 for realizing the ideal protocol $SC(G)$ in UC#. Instead, we have to consider an appropriate ideal functionality along with an appropriate ideal protocol.

Let $\Pi^*_G$ be the protocol of Construction 1 for realizing $AG([G])$ for some ideal functionality $G$. We consider the protocol $\Pi^*_G$ that is identical to $\Pi_G$, except that all un hackable sub-parties are now hackable and that all enhanced channels are replaced by standard channels. As a consequence, all main parties and their sub-parties, including e.g. the previously un hackable Enc unit or the OIM, are always online and susceptible to online attacks.

As $\Pi^*_G$ still makes use of a (now hackable) interface module, it is not a legal protocol in the standard UC framework. For the same reason, we cannot hope to prove that $\Pi^*_G$ realizes the ideal protocol $SC(G)$ in UC#, i.e. provides standard adaptive UC security. Instead, we have to consider an appropriate ideal functionality along with an appropriate ideal protocol.

PID-wise Corruption

We now discuss the case of PID-wise corruption, i.e. the case where the adversary may only corrupt a main party and its sub-parties together and defer more fine-grained corruption models to the full version. In the setting of PID-wise corruption, it suffices to consider the ideal functionality $G'$ that is identical to $G$, except that it provides the output of parties via their respective dummy OIM. The corresponding ideal protocol is identical to $SC(G)$, except that for each party $P_i$, there also exists a (hackable) dummy OIM connected to the environment and $G'$ via standard channels. As the sub-protocol $F_G$ is adaptively UC-secure, it can be shown that $\Pi^*_G$ securely realizes $SC'(G')$ in the UC# framework.

B Proof of Theorem 1

We will prove that $\Pi_G$ emulates the ideal protocol $AG([G])$ in the UC# framework for adversaries corrupting at most $N - 1$ parties $P \in \{P_1, \ldots, P_N\}$ under the assumptions that $PKE$ is IND-parallel-CCA-secure, $SIG$ is EUF-naCMA-secure and length-normal and $MAC$ is EUF-1-CMA-secure.

Next, we define the following experiment, which will be used in the proof to show that an environment $Z$ cannot send “fake messages” $(P_i, c')$ to an honest party $P_i$ as well as their sub-parties together and defer more fine-grained UC security.

**Definition 4 (Auxiliary Experiment).** The experiment $Exp_{A(z), PKE, SIG}^{aux}(n)$ is defined as follows:

At the beginning, the experiment generates keys $(pk, sk) \leftarrow Gen_{PKE}(1^n)$ and $(vk, sgk) \leftarrow Gen_{SIG}(1^n)$. On input $1^n$, $z$ and $pk$, the adversary $A$ may then non-adaptively send queries to a signing oracle $O_{Sig(sgk)}$. Afterwards, the experiment sends $vk$ to $A$. $A$ may then send a message of the form $(prf_1, prf_2, m)$ to the experiment. The experiment then computes $\sigma \leftarrow Sig(sgk, (prf_1, m))$, $c^* \leftarrow Enc(pk, (prf_1, m, \sigma))$, and sends $c^*$ to $A$. During the experiment, $A$ may send a single parallel query to a decryption oracle $O_{Dec(sgk)}$ subject to the restriction that the query does not contain $c^*$. At the end of the experiment, $A$ sends a tuple $(m', \sigma')$ to the experiment. The experiment then checks if $Vrfy_{SIG}(vk, m', \sigma') = 1$ and $m'$ has not been sent to $O_{Sig(sgk)}$ before. If this holds, the experiment outputs 1 and 0 otherwise.

We have the following lemma.

**Lemma 1.** If $PKE$ is IND-pCCA-secure and $SIG$ EUF-naCMA-secure, then for every PPT adversary $A$ and all $z \in \{0, 1\}^*$, there exists a negligible function $\text{negl}$ such that

$$\Pr[Exp_{A(z), PKE, SIG}^{aux}(n) = 1] \leq \text{negl}(n).$$

**Sketch.** Assume there exists an adversary $A$ that wins in the experiment $Exp_{PKE, SIG, A(z)}^{aux}(n)$ with non-negligible probability. Since $PKE$ is IND-pCCA-secure, one can replace $c^*$ by $c' \leftarrow Enc(pk, 0^l)$, where $L = |(prf_1, m, \sigma)|$, incurring only a negligible loss in $A$’s success probability. Then, one can directly construct an adversary $A'$ out of $A$ that breaks the EUF-naCMA-security of $SIG$ with non-negligible probability. $A'$ simply internally simulates the experiment $Exp_{PKE, SIG, A(z)}^{aux}(n)$ for $A$ using his signing oracle and $c'$ for $c'$. Once $A$ sends a tuple $(m, \sigma)$ to the experiment, $A'$ sends $(m, \sigma)$ to the EUF-naCMA experiment. $A'$ then wins in the
EUF-naCMA experiment if and only if $A$ wins in the experiment $\text{Exp}_{\text{PKE,SIG},A(\cdot)}^{\text{aux}}(n)$.

Next, we define the simulator for the dummy adversary.

**Definition 5** (Definition of the Simulator). Define the simulator $\text{Sim}$ interacting with an environment $Z$ and a fortified ideal functionality $[G]$ as follows:

1. At the beginning, $\text{Sim}$ internally defines $N$ parties corresponding to the parties in $\Pi$. Throughout the simulation, $\text{Sim}$ will keep track of the online state of these parties by marking them as **online** or **offline**.
   - At the beginning, $\text{Sim}$ marks these parties according to the initial online states of the dummy parties in the ideal protocol.
2. $\text{Sim}$ initializes a Boolean variable $\text{verify} = \text{true}$.
3. $\text{Sim}$ carries out the physical-attack instruction received from $Z$ on its first activation. $\text{Sim}$ carries out an (online-attack, $P_i$) instruction only if $\text{Sim}$ has marked party $P_i$ as **online**.
4. Each time $Z$ sends status, $\text{Sim}$ sends the set of markings of each party.
5. Throughout the simulation, $\text{Sim}$ reports the respective notify transport tokens to $Z$ (note that we will not mention them anymore in the following).
6. $\text{Sim}$ generates $(\text{pk}_i, \text{sk}_i) \leftarrow \text{Gen}_{\text{PKE}}(1^n)$, $k_i \leftarrow \text{Gen}_{\text{MAC}}(1^n)$ and $(\text{sgk}_i, \text{vk}_i) \leftarrow \text{Gen}_{\text{SIG}}(1^n)$ for each party $P_i$ that was not corrupted before receiving input.
7. For each $i$ such that party $P_i$ is honest, $\text{Sim}$ reports (registered, $\text{sid}'_i$, $\text{RM}_i$, $\text{pk}_i$, $\text{vk}_i$). If $Z$ answers with “ok”, $\text{Sim}$ stores $(\text{pk}_i, \text{vk}_i)$ as “registered”.
8. Each time $\text{Sim}$ is activated by $[G]$ after an honest party $P_i$ received its input, $\text{Sim}$ generates $3N$ random strings $s'_{ij}, r'_{ij}, k'_{ij}$, computes $\sigma'_{ij} \leftarrow \text{Sig}(\text{sgk}_i, (P_i, s'_{ij}, r'_{ij}, k'_{ij}))$ ($j = 1, \ldots, N$) and $c_{ij} \leftarrow \text{Enc}(\text{pk}_j, (P_i, s'_{ij}, r'_{ij}, k'_{ij}, \sigma'_{ij}))$. Each time party $Z$ activates the Enc unit of $\text{Sim}$, $\text{Sim}$ reports the respective tuple $(P_i, c_{ij})$ if $\text{pk}_j$ is stored as “registered”.
9. Once $\text{Sim}$ has reported all $(P_i, c_{ij})$ for $j = 1, \ldots, N$ as well as (registered, $\text{sid}'$, $\text{RM}_i$, $\text{pk}_i$, $\text{vk}_i$) for an honest $P_i$, $\text{Sim}$ marks $P_i$ as **online**.
10. If a party $P_i$ is corrupted after receiving input, $\text{Sim}$ sends $(s'_{ii}, r'_{ii}, k'_{ii}, \sigma'_{ii}, \text{vk}_i, \text{sk}_i)$ to $Z$.
11. If $Z$ sends a value $(\text{pk}_j, \text{vk}_j)$ to $\mathcal{F}_{\text{reg}}$ for a party $P_i$ corrupted before receiving input, $\text{Sim}$ stores $(\text{pk}_j, \text{vk}_j)$ as “registered”.
12. Each time $Z$ sends a message addressed to buffer of a party $P_i$, $\text{Sim}$ stores that message as a message “received by $P_i$”.
13. If $Z$ activates an honest party $P_j$ who is marked as **online** and has received at least $N - 1$ messages and all $\text{vk}_l$ ($l = 1, \ldots, N$) are stored as “registered”, then $\text{Sim}$ stores $\text{vk}_j = (\text{vk}_1, \ldots, \text{vk}_N)$ and reports (received, $P_i$) to $Z$. Upon receiving (confirmed, $P_i$) from $Z$, $\text{Sim}$ marks $P_j$ as input given.
14. If $Z$ sends a tuple consisting of a vector $\text{vk}_j$ and $(s'_{ij}, r'_{ij}, k'_{ij}, \sigma'_{ij})$ ($l = 1, \ldots, N$) as the input to $\mathcal{F}_G$ for a corrupted party $P_i$, then $\text{Sim}$ stores that input (if an input has already been stored for $P_j$ then $\text{Sim}$ overwrites it) and, if not done yet, marks $P_j$ as input given.
15. Once all parties are marked as input given, $\text{Sim}$ does the following:
   
   (i) If not all $\text{vk}_i$ ($i = 1, \ldots, N$) are equal, $\text{Sim}$ sets $\text{verify} = \text{false}$.
   
   (ii) For each $j$ such that $P_j$ is honest, $\text{Sim}$ checks the following conditions:
      
      - $P_j$ has received for each $i$ such that party $P_i$ is not corrupted before receiving input, $\text{Sim}$ reports (registered, $\text{sid}'_i$, $\text{RM}_i$, $\text{pk}_i$, $\text{vk}_i$).
      - $P_j$ has received for each $l$ such that party $P_l$ is corrupted before receiving input a set $M_l$ fulfilling property (⋆) from p. 324.
      
      If at least one of these conditions does not hold, $\text{Sim}$ sets $\text{verify} = \text{false}$.
   
   (iii) For each tuple consisting of a vector $\text{vk}_j$ and $(s'_{ij}, r'_{ij}, k'_{ij}, \sigma'_{ij})$ ($l = 1, \ldots, N$) which was stored by $\text{Sim}$ as the input to $\mathcal{F}_G$ for a corrupted party $P_j$, $\text{Sim}$ checks the following:
      
      - for each $i$ such that party $P_i$ was not corrupted before receiving input, $\text{Sim}$ checks if $(s'_{ij}, r'_{ij}, k'_{ij}) = (s_{ij}, r_{ij}, k_{ij})$, where $(s_{ij}, r_{ij}, k_{ij})$ is the respective tuple generated by $\text{Sim}$. If this does not hold or $\text{Vrfy}_{\text{SIG}}(\text{vk}_i, (P_j, s'_{ij}, r'_{ij}, k'_{ij}), \sigma'_{ij}) = 0$, $\text{Sim}$ sets $\text{verify} = \text{false}$.
      - for each $l$ such that party $P_l$ was corrupted before receiving input, $\text{Sim}$ sets $\text{verify} = \text{false}$ if $\text{Vrfy}_{\text{SIG}}(\text{vk}_l, (P_j, s'_{ij}, r'_{ij}, k'_{ij}), \sigma'_{ij}) = 0$.

16. $\text{Sim}$ extracts the input, MAC key and random pad of each party $P_i$ corrupted before receiving input by decrypting all ciphertexts coming from $Z$, and by observing the inputs $Z$ sends to $\mathcal{F}_G$ for corrupted parties. $\text{Sim}$ sends each extracted input to $[G]$.
17. Once all parties are marked as input given and $Z$ activates an honest party $P_i$, then $\text{Sim}$ instructs $[G]$ to
(i) send the output to $P_i$'s dummy OIM if verify = true.
(ii) output ⊥ to $P_i$'s dummy OIM if verify = false.

18. Once all parties are marked as input given and $Z$ requests the output of $F_G$ for a party $P_i$ corrupted after receiving input, then
(i) If verify = true, Sim generates a random string $y_i \leftarrow \{0,1\}^{p_i(n)}$ and sends $(y_i, \text{Mac}(k_i, y_i))$ to $Z$.
(ii) If verify = false, Sim sends ⊥ to $Z$.

19. If $Z$ sends a message $(m', t')$ addressed to OIM of a party $P_i$ corrupted after receiving input, then
(i) If $Z$ has not yet requested the output of $F_G$ for $P_i$, yet, Sim instructs $[G]$ to output ⊥ to the dummy OIM of $P_i$.
(ii) If $Z$ has already requested the output of $F_G$ for $P_i$ and Sim sent $(y_i, \text{Mac}(k_i, y_i))$ (in item 18) to $Z$, then
   - If $m' = y_i$, Sim instructs $[G]$ to output ⊥ to the dummy OIM of $P_i$.
   - If $m' \neq y_i$ and Vrfy$_{MAC}(k_i, m', t') = 1$, then Sim instructs $[G]$ to send the output to the dummy OIM of party $P_i$. Otherwise, Sim instructs $[G]$ to output ⊥ to the dummy OIM of $P_i$.
(iii) If $Z$ has already requested the output of $F_G$ for $P_i$ and Sim sent ⊥ (in item 18) to $Z$, then Sim instructs $[G]$ to output ⊥ to $P_i$'s dummy OIM.

20. Once all parties are marked as input given and $Z$ requests the output of $F_G$ for a party $P_i$ corrupted before receiving input, then
(i) If verify = true, Sim sends $(y_i + r_i, \text{Mac}(k_i, y_i + r_i))$ to $Z$, where $y_i$ is the output of $[G]$ for party $P_i$ and $k_i, r_i$ are the MAC key and random pad extracted in item 16.
(ii) If verify = false, Sim sends ⊥ to $Z$.

21. Sim lets $Z$ determine the output of the dummy OIM of each party corrupted before receiving input.

Proof. It suffices to find a simulator for the dummy adversary. In the following proof, we will consider a sequence of hybrids $H_0, \ldots, H_4$. Starting from the real protocol $\Pi_G$, we will define ideal protocols that gradually reduce the simulator's abilities (i.e. restrict the set of parties for which he may learn/modify the inputs/outputs). The final hybrid $H_4$ will be the ideal protocol $\mathcal{A}_G([G])$ and the simulator as defined in Definition 5.

Let $Z$ be an environment that instructs $D$ to corrupt at most $N - 1$ parties $P \in \{P_1, \ldots, P_N\}$. Let $\text{out}_i(Z)$ be the output of $Z$ in the hybrid $H_i$. In the following, we will say corrupted before input and corrupted after input for brevity.

**Hybrid $H_0$**

Let $H_0$ be the execution experiment between the environment $Z$, the dummy adversary $D$ and the real protocol $\Pi_G$.

**Hybrid $H_1$**

Let $H_1$ be the execution experiment between the environment $Z$, the ideal protocol $\mathcal{A}_G([F_1])$ and the ideal-model adversary $\text{Sim}_1$, where $F_1$ and $\text{Sim}_1$ are defined as follows: Define $F_1$ to be identical to $[G]$ except for the following: $F_1$ hands the adversary the inputs and outputs of every party (honest and corrupted) and allows him to determine the outputs of the dummy OIMs of all corrupted parties (i.e. all parties corrupted before and after input). Note that, like $[G]$, $F_1$ does not allow the adversary to modify the inputs of parties corrupted after input (unless all parties are corrupted) and also does not allow him to modify the inputs of honest parties.

Define $\text{Sim}_1$ to be like the simulator in Definition 5 except for the following: In item 8, $\text{Sim}_1$ reports the ciphertexts as they are generated in the real protocol (in particular, generates shares of the actual inputs). Also, if a party $P_i$ is corrupted after having received input, $\text{Sim}_1$ reports the respective shares as they are generated in the real protocol in item 10 along with a valid signature and $vk_i, sk_i$. In item 18, if verify = true, $\text{Sim}_1$ reports $(y_i + r_i, \text{Mac}(k_i, y_i + r_i))$ to $Z$, where $y_i$ is the output $\text{Sim}_1$ receives for the respective party from $F_1$ and $k_i, r_i$ are the MAC key and one-time pad generated in items 6 and 8. If verify = false, $\text{Sim}_1$ reports ⊥. In item 19, if $Z$ sends a message $(m', t')$ addressed to OIM of a party $P_i$ (corrupted after input), $\text{Sim}_1$ carries out the program of the OIM (using the MAC key and one-time pad generated in items 6 and 8), computing a value $y' \in \{0,1\}^{p_i(n)} \cup \{\bot\}$, and then instructs $[G]$ to output $y'$ to $P_i$'s dummy OIM.

Consider the following events:

Let $E_{\text{fake}}$ be the event that there exists an honest party $P_j$ that retrieves a tuple $(P_i, c')$ in its buffer such that party $P_i$ is not corrupted before input and $(P_i, c')$ is “valid”, i.e. Dec$(sk_j, c') = (P_i, s'_{ij}, r'_{ij}, k'_i, \sigma'_{ij})$ and Vrfy$_{SIG}(vk_j, (P_j, s'_{ij}, r'_{ij}, k'_i), \sigma'_{ij}) = 1$, but either $c' \neq c_{ij}$ or $c_{ij}$ has not been generated yet (by the Enc unit of party $P_i$).

Let $E_{\text{fake}}$ be the event that $Z$ sends an input $(s'_{ij}, r'_{ij}, k'_i, \sigma'_{ij})$ for a corrupted party $P_j$ to
\( \mathcal{F}_g \) such that party \( P_i \) is not corrupted before input and \( \text{Vrfy}_{\text{SIG}}(vk_i, (P_j, s'_{ij}, r'_{ij}, k'_ij), \sigma'_{ij}) = 1 \), but \((s'_{ij}, r'_{ij}, k'_ij) \neq (s_{ij}, r_{ij}, k_{ij}) \) (where \( vk_i \) and \( (s_{ij}, r_{ij}, k_{ij}) \) were generated by \( P_i \)).

Let \( E = E_{\text{fakemess}} \cup E_{\text{fakein}} \). It holds that
\[
\Pr[\text{out}_0(Z) = 1 \land \neg E] = \Pr[\text{out}_1(Z) = 1 \land \neg E].
\]

This is because if \( E_{\text{fakemess}} \) does not occur then a message in the buffer of a party \( P_j \) that is addressed as coming from a party \( P_i \) who was not corrupted before input decrypts to a valid message/signature pair if and only if it equals the ciphertext \( c_{ij} \) sent by \( P_i \). Moreover, for each corrupted party \( P_i \), since \( E_{\text{fakein}} \) does not occur, \( Z \) only sends inputs \((s'_{ij}, r'_{ij}, k'_ij, \sigma'_{ij}) \) to \( \mathcal{F}_g \) such that either \( \text{Vrfy}_{\text{SIG}}(vk_i, (P_j, s'_{ij}, r'_{ij}, k'_ij), \sigma'_{ij}) = 0 \) or \( \text{Vrfy}_{\text{SIG}}(vk_i, (P_j, s'_{ij}, r'_{ij}, k'_ij), \sigma'_{ij}) = 1 \) and \((s'_{ij}, r'_{ij}, k'_ij) = (s_{ij}, r_{ij}, k_{ij}) \) was generated by party \( P_i \) (who was not corrupted before input).

Therefore, it holds that
\[
|\Pr[\text{out}_0(Z) = 1] - \Pr[\text{out}_1(Z) = 1]| \leq \Pr[E] \\
\leq \Pr[E_{\text{fakemess}}] + \Pr[E_{\text{fakein}}].
\]

**Claim 1:** \( \Pr[E_{\text{fakemess}}] \) is negligible.

Consider the following adversary \( A \) in the auxiliary experiment \( \text{Exp}_{\text{PKESIG,A}(z)}^{\text{aux}}(n) \): At the beginning, \( A \) randomly selects a tuple \((i, j) \in \{1, \ldots, N\} \times \{1, \ldots, N\} \) such that \( i \neq j \). \( A \) then simulates hybrid \( H_0 \) using the public key \( pk_j \) from the experiment for \( pk_j \) in its internal simulation. When \( Z \) gives the party \( P_i \) its input \( x_i \), \( A \) generates shares \( s_{ij}, r_{ij}, k_{ij} \) of \( x_i \), of a random pad \( r_i \) and of a MAC key \( k_i \) just like in \( H_0 \). \( A \) sends the tuples \((P_i, s_{ij}, r_{ij}, k_{ij}) \) for \( l \neq j \) to the signing oracle \( \mathcal{O}_{\text{SIG}(sk_j)} \), receiving signatures \( \sigma_{ij} \). After receiving the verification key \( vk \) from the experiment, \( A \) uses \( vk \) for \( vk_i \) in its internal simulation. Using \( pk \), \( A \) encrypts all tuples \((P_i, s_{ij}, r_{ij}, k_{ij}) \) (\( l \neq \{i, j\} \)) and sends them to the respective party in its internal simulation. Once the message \((P_i, c_{ij}) \) is supposed to be sent in the internal simulation, \( A \) sends \((P_i, P_j, s_{ij}, r_{ij}, k_{ij}) \) to the experiment, receiving \( c^* \). \( A \) then uses \((P_i, c^*) \) for \((P_i, c_{ij}) \) in its simulation. When \( P_j \) is activated and is online and has received at least \( N - 1 \) messages, \( A \) sends all ciphertexts addressed as coming from \( P_i \) such that \( c \neq c^* \) to the decryption oracle \( \mathcal{O}_{\text{Dec}(sk_j)} \). If \( c^* \) has not been generated yet, \( A \) sends all ciphertexts addressed as coming from \( P_i \). For each message \((P_i, m, \sigma) \) he receives from the oracle \( \mathcal{O}_{\text{Dec}(sk_j)} \), \( A \) checks if \( \text{Vrfy}_{\text{SIG}}(vk, (P_j, m), \sigma) = 1 \). If this holds for a message \((P_i, m', \sigma') \), then \( A \) sends \((P_j, m', \sigma') \) to the experiment. If during the simulation, \( P_i \) is corrupted before input or \( P_j \) is corrupted (before or after input) or if no message \( A \) receives from \( \mathcal{O}_{\text{Dec}(sk_j)} \), then \( A \) sends \( \bot \) to the experiment.

By construction, it holds that if \( E_{\text{fakemess}} \) occurs and \( A \) has correctly guessed an index \((i, j) \) for which \( \text{Vrfy}_{\text{SIG}} \) occurs, then \( A \) sends a message \( c' \) to \( \mathcal{O}_{\text{Dec}(sk_j)} \) such that \( c \neq c^* \) or \( c^* \) has not been generated yet and \( \text{Dec}(sk, c') = (P_i, m', \sigma') \) and \( \text{Vrfy}_{\text{SIG}}(vk, (P_j, m', \sigma')) = 1 \). Since \( A \) does not send a message of the form \((P_j, m)\) to the signing oracle \( \mathcal{O}_{\text{SIG}(sk_j)} \), it follows that \( \text{Exp}_{\text{PKESIG,A}(z)}^{\text{aux}}(n) = 1 \).

Furthermore, the probability that \( A \) correctly guesses an index \((i, j) \) for which \( E_{\text{fakemess}} \) occurs is at least \( 1/(N \cdot (N - 1)) \). Hence,
\[
\Pr[\text{Exp}_{\text{PKESIG,A}(z)}^{\text{aux}}(n) = 1] \geq \Pr[E_{\text{fakemess}}]/(N \cdot (N - 1)).
\]

Therefore, since \( \Pr[\text{Exp}_{\text{PKESIG,A}(z)}^{\text{aux}}(n) = 1] \) is negligible by Lemma 1 and \( N \cdot (N - 1) \) is polynomial in \( n \), it follows that \( \Pr[E_{\text{fakemess}}] \) is also negligible.

**Claim 2:** \( \Pr[E_{\text{fakein}}] \) is negligible.

Consider the following adversary \( A \) against the EUF-naCMA security of \( \text{SIG} \): At the beginning, \( A \) randomly selects an index \( i \in \{1, \ldots, N\} \), then \( A \) simulates hybrid \( H_0 \). When \( Z \) gives the party \( P_i \) its input \( x_i \), \( A \) generates shares \( s_{ij}, r_{ij}, k_{ij} \) of \( x_i \), of a random pad \( r_i \) and of a MAC key \( k_i \) just like in \( H_0 \). \( A \) sends the tuples \((P_j, s_{ij}, r_{ij}, k_{ij}) \) to the signing oracle \( \mathcal{O}_{\text{SIG}(sk_j)} \), receiving signatures \( \sigma_{ij} \). After receiving \( vk \), \( A \) then uses \( vk \) for \( vk_i \), encrypts all tuples \((P_i, s_{ij}, r_{ij}, k_{ij}, \sigma_{ij}) \) (\( j = 1, \ldots, N \)) and sends them to the respective party in its internal simulation. Each time \( Z \) sends a tuple \((s'_{ij}, r'_{ij}, k'_ij, \sigma'_{ij}) \) as input for a corrupted party \( P_j \) to \( \mathcal{F}_g \) such that \((s'_{ij}, r'_{ij}, k'_ij) \neq (s_{ij}, r_{ij}, k_{ij}) \), \( A \) checks if \( \text{Vrfy}_{\text{SIG}}(vk_i, (P_j, s'_{ij}, r'_{ij}, k'_ij), \sigma'_{ij}) = 1 \). If this holds, \( A \) sends \((P_j, s'_{ij}, r'_{ij}, k'_ij, \sigma'_{ij}) \) to the experiment. If during the simulation, \( P_i \) is corrupted before input or no message \( A \) checks is valid, then \( A \) sends \( \bot \) to the experiment.

By construction, it holds that if \( E_{\text{fakein}} \) occurs and \( A \) has correctly guessed an index \( i \) for which \( E_{\text{fakein}} \) occurs, then \( \text{Exp}_{\text{PKESIG,A}(z)}^{\text{aux}}(n) = 1 \) because the tuple \((P_j, s'_{ij}, r'_{ij}, k'_ij, \sigma'_{ij}) \) is valid and \((P_j, s'_{ij}, r'_{ij}, k'_ij) \neq (P_i, s_{ij}, r_{ij}, k_{ij}) \) has not been sent to the signing oracle \( \mathcal{O}_{\text{SIG}(sk_j)} \). Furthermore, the probability that \( A \) correctly guesses an index \( i \) for which \( E_{\text{fakein}} \) occurs is at least \( 1/N \). Hence,
\[
\Pr[\text{Exp}_{\text{SIG},A}(z), (n) = 1] \geq \Pr[\text{E}_{\text{fakein}}]/N.
\]

Therefore, since \( \Pr[\text{Exp}_{\text{SIG},A}(z), (n) = 1] \) is negligible because SIG is EUF-naCMA-secure by assumption and \( N \) is polynomial in \( n \), it follows that \( \Pr[\text{E}_{\text{fakein}}] \) is also negligible. Hence, there exist a negligible function \( \text{neg}_1 \) such that
\[
|\Pr[\text{out}_0(Z) = 1] - \Pr[\text{out}_1(Z) = 1]| \leq \text{neg}_1(n).
\]

**Hybrid \( H_2 \)**

Let \( H_2 \) be the execution experiment between the environment \( Z \), the ideal protocol \( \mathcal{AG}(F_1) \) (again) and the ideal-model adversary \( \mathcal{Sim}_2 \), where \( \mathcal{Sim}_2 \) is defined as follows:

Define \( \mathcal{Sim}_2 \) to be like \( \mathcal{Sim}_1 \) except for the following: In item 8, each time \( \mathcal{Sim}_2 \) is activated by \( F_1 \) after an honest party \( P_i \) received its input, \( \mathcal{Sim}_2 \) generates \( N \) random strings \( k'_{ij} \) and computes \( \sigma'_{ij} \leftarrow \text{Sig}(\text{sgk}_{ij}(P_j, s_{ij}, r_{ij}, k'_{ij})) \) (\( j = 1, \ldots, N \)), where the \( s_{ij} \) and \( r_{ij} \) (\( j = 1, \ldots, N \)) are still the shares of the input \( x_i \) and a random pad \( r_i \), respectively. \( \mathcal{Sim}_2 \) then iteratively reports \( (P_i, \text{Enc}(\text{pk}_{ij}(P_i, s_{ij}, r_{ij}, k'_{ij}, \sigma'_{ij}))) \) (\( j \in \{1, \ldots, N\} \setminus \{i\} \)) to \( Z \). If a party \( P_i \) is corrupted after having received input, \( \mathcal{Sim}_2 \) sends \( (s_{ii}, r_{ii}, k'_{ii}, \sigma'_{ii}, v_{ki}, s_{ki}) \) to \( Z \) in item 10. (Note that in item 18 \( \mathcal{Sim}_2 \) still uses the MAC key \( k_i \leftarrow \text{GenMAC}(1^n) \) generated in item 6 for the output of \( F_G \) to a party \( P_i \) corrupted after input (if that output is \( \neq \bot \))).

Let \( H_{2,0}, \ldots, H_{2,N} \) be the execution experiment between the environment \( Z \), the ideal protocol \( \mathcal{AG}(F_1) \) and the ideal-model adversary \( \mathcal{Sim}_{2,0}, \ldots, \mathcal{Sim}_{2,N} \), respectively, where \( \mathcal{Sim}_{2,i} \) is defined as follows:

Define the simulators \( \mathcal{Sim}_{2,i} \) to be like \( \mathcal{Sim}_1 \) except for the following: In item 8, each time \( \mathcal{Sim}_{2,i} \) is activated by \( F_1 \) after an honest party \( P_i \) \( \in \{P_1, \ldots, P_i\} \) received its input, \( \mathcal{Sim}_{2,i} \) generates \( N \) random strings \( k'_{ij} \), computes \( \sigma'_{ij} \leftarrow \text{Sig}(\text{sgk}_{ij}(P_j, s_{ij}, r_{ij}, k'_{ij})) \) (\( j = 1, \ldots, N \)), and iteratively reports \( (P_i, \text{Enc}(\text{pk}_{ij}(P_i, s_{ij}, r_{ij}, k'_{ij}, \sigma'_{ij}))) \) (\( j \in \{1, \ldots, N\} \setminus \{l\} \)) to \( Z \). If a party \( P_i \) \( \in \{P_1, \ldots, P_i\} \) is corrupted after having received input, \( \mathcal{Sim}_{2,i} \) sends \( (s_{ij}, r_{ij}, k'_{ij}, \sigma'_{ij}, v_{ki}, s_{ki}) \) to \( Z \) in item 10.

It holds that
\[
\Pr[\text{out}_{2,0}(Z) = 1] = \Pr[\text{out}_{1}(Z) = 1]
\]

and
\[
\Pr[\text{out}_{2,N}(Z) = 1] = \Pr[\text{out}_{2}(Z) = 1].
\]

Assume that there exists a non-negligible function \( \epsilon \) such that \( \Pr[\text{out}_1(Z) = 1] = \Pr[\text{out}_2(Z) = 1] > \epsilon \). Then there exists an \( i^* \in \{1, \ldots, N\} \) such that
\[
|\Pr[\text{out}_{2,i^*-1}(Z) = 1] - \Pr[\text{out}_{2,i^*}(Z) = 1]| > \epsilon/N.
\]

Moreover, if party \( P_i^* \) is not corrupted after input, i.e. if it is corrupted before input or remains honest throughout the execution, then the views of \( Z \) in \( H_{2,i^*-1} \) and \( H_{2,i^*} \) are identically distributed. Therefore,
\[
\epsilon/N < |\Pr[\text{out}_{2,i^*-1}(Z) = 1] - \Pr[\text{out}_{2,i^*}(Z) = 1]| = |\Pr[\text{out}_{2,i^*-1}(Z) = 1] - \Pr[\text{out}_{2,i^*}(Z) = 1]|
\]

\( \wedge P_{i^*} \text{ corrupted after input} \]
\[
\neg \Pr[\text{out}_{2,i^*}(Z) = 1]
\]

\( \wedge P_{i^*} \text{ corrupted after input} \]
the view of $Z$ in $\mathcal{A}$'s internal simulation is distributed as in the experiment $H_{2,i\cdot}$. Moreover, assuming party $P_i$ is corrupted after input, the probability that $\mathcal{A}$ guesses an index $j$ such that party $P_j$ remains honest is at least $1/(N-1)$. Hence,

$$\Pr[\text{output}_0(\mathcal{A}) = 1] - \Pr[\text{output}_1(\mathcal{A}) = 1]$$

$$= \Pr[\text{output}_{2,\cdot 1}(Z) = 1 \land P_i \text{ corrupted after input} \wedge \text{Guess correct}] - \Pr[\text{output}_{2,\cdot 1}(Z) = 1 \land P_i \text{ corrupted after input} \wedge \text{Guess correct}]$$

$$> \epsilon/(N \cdot (N-1)).$$

This contradicts the IND-pCCA security of PKE. Hence, there exist a negligible function $\text{negl}_2$ such that

$$|\Pr[\text{output}_1(Z) = 1] - \Pr[\text{output}_2(Z) = 1]| \leq \text{negl}_2(n).$$

**Hybrid $H_3$**

Let $H_3$ be the execution experiment between the environment $Z$, the ideal protocol $AG(F_2)$ and the ideal-model adversary $Sim_3$, where $F_2$ and $Sim_3$ are defined as follows:

Let $F_2$ be identical to $F_1$ except that now the adversary is allowed to determine the outputs only of the dummy OIMs of the parties corrupted before input.

Define $Sim_3$ to be like $Sim_2$ except that item 19 is identical to the same step of the simulator in Definition 5. Let $E_{\text{fakeout}}$ be the event that $Z$ sends a message $(m', t')$ to OIM of a party $P_i$ corrupted after input such that $\text{Vrfy}_{\text{MAC}}(k, m', t') = 1$ but either $P_i$ has received $\perp$ from $F_2$ or a tuple $(m, t)$ such that $m' \neq m$ or $P_i$ has not received an output from $F_2$ yet. It is easy to see that the following holds:

$$\Pr[\text{output}_2(Z) = 1 \land \neg E_{\text{fakeout}}] = \Pr[\text{output}_3(Z) = 1 \land \neg E_{\text{fakeout}}].$$

Therefore, it holds that

$$|\Pr[\text{output}_2(Z) = 1] - \Pr[\text{output}_3(Z) = 1]| \leq \Pr[E_{\text{fakeout}}].$$

**Claim 3: $\Pr[E_{\text{fakeout}}]$ is negligible**

Consider the adversary $\mathcal{A}$ against the EUF-1-CMA-security of MAC. At the beginning, $\mathcal{A}$ randomly selects an index $i \in \{1, \ldots, N\}$. $\mathcal{A}$ then simulates the hybrid $H_2$. Once $Z$ expects the output from $F_2$ for party $P_i$ (if $P_i$ is corrupted after input), $\mathcal{A}$ computes the (padded) result $m$ for this party. If $m = \perp$, $\mathcal{A}$ sends $\perp$ to $Z$. Otherwise, $\mathcal{A}$ sends $m$ to the MAC oracle $O_{\text{Mac}(k, \cdot)}$, receiving a tag $t$. $\mathcal{A}$ then sends $(m, t)$ to $Z$. If $Z$ sends a tuple $(m', t')$ to OIM of $P_i$ such that $m' \neq m$, then $\mathcal{A}$ sends $(m', t')$ to the experiment. If during the simulation, $P_i$ is not corrupted after input or if $Z$ sends $\perp$ or a tuple $(m', t')$ such that $m' = m$ to OIM of $P_i$, then $\mathcal{A}$ sends $\perp$ to the experiment.

By construction, it holds that if $E_{\text{fakeout}}$ occurs and $\mathcal{A}$ correctly guessed an index for which $E_{\text{fakeout}}$ occurs, then $\text{Exp}_{\text{MAC}; A}(n) = 1$ because $(m', t')$ is valid and $m' \neq m$ has not been sent to $O_{\text{Mac}(k, \cdot)}$. Moreover, the probability that $\mathcal{A}$ correctly guesses an index for which $E_{\text{fakeout}}$ occurs is at least $1/N$. Hence,

$$\Pr[\text{Exp}_{\text{MAC}; A}(n) = 1] = \Pr[E_{\text{fakeout}}]/N.$$

Therefore, since $\Pr[\text{Exp}_{\text{MAC}; A}(n) = 1]$ is negligible because MAC is EUF-1-CMA-secure by assumption and $N$ is polynomial in $n$, it follows that $\Pr[E_{\text{fakeout}}]$ is also negligible. Hence, there exist a negligible function $\text{negl}_3$ such that

$$|\Pr[\text{output}_2(Z) = 1] - \Pr[\text{output}_3(Z) = 1]| \leq \text{negl}_3(n).$$

**Hybrid $H_4$**

Let $H_4$ be the execution experiment between $Z$, the ideal protocol $AG(F_3)$ and the ideal-model adversary $Sim_4$, where $F_3$ and $Sim_4$ are defined as follows: Let $F_3$ be identical to $F_2$ except that the adversary is not given the inputs and outputs of honest parties anymore. The adversary is only given the inputs and outputs of parties corrupted after input when all parties are corrupted.

Define the adversary $Sim_4$ to be like $Sim_3$ except that items 8, 10 and 18 are identical to the same steps of the simulator in Definition 5.

Using an argument that is almost identical to the one in hybrid $H_2$ one can show that there exists a negligible function $\text{negl}_4$ such that

$$|\Pr[\text{output}_3(Z) = 1] - \Pr[\text{output}_4(Z) = 1]| \leq \text{negl}_4(n).$$

Since $H_4$ is identical an execution between $Z$, the ideal protocol $AG([G])$ and the simulator as defined in Definition 5, it follows that there exists a negligible function $\text{negl}_5$ such that

$$|\Pr[\text{Exec}_{UC}^G ([G], D, Z) = 1] - \Pr[\text{Exec}_{UC}^G (AG([G]), \text{Sim}, Z) = 1]| \leq \text{negl}_5(n).$$

The statement follows. \qed
C A Short Introduction to the UC Framework

In the following, we give a brief overview of the UC framework. The following is adapted from [10]. For a detailed introduction see [11].

In the UC framework, security is defined by the indistinguishability of two experiments: the ideal experiment and the real experiment. In the ideal experiment, the task at hand is carried out by dummy parties with the help of an ideal incorruptible entity—called the ideal functionality $F$. In the real experiment, the parties execute a protocol $\pi$ in order to solve the prescribed task themselves. A protocol $\pi$ is said to be a (secure) realization of $F$ if no PPT machine $Z$, called the environment, can distinguish between these two experiments. In contrast to previous simulation-based notions, indistinguishability must not only hold after the protocol execution has completed, but even if the environment $Z$—acting as the interactive distinguisher—takes part in the experiment, orchestrates all adversarial attacks, gives input to the parties running the challenge protocol, receives the parties’ output and observes the communication during the whole protocol execution.

The Basic Model of Computation

The basic model of computation consists of a set of (a polynomial number of) instances (ITIs) of interactive Turing machines (ITMs). An ITM is the description of a Turing machine with an additional identity tape, three externally writable input tapes (namely for input, subroutine output and incoming messages) and an outgoing message tape. The latter is jointly used to provide input to any of the three input tapes of another ITM. The tangible instantiation of an ITM—the ITI—is identified by the content of its identity tape. The order of activation of the ITIs is completely asynchronous and message-driven. An ITI gets activated if input, subroutine output or an incoming message is written onto its respective tape. If the ITI writes onto its outgoing message tape and calls the special external write instruction, the activation of this ITI completes. The message must explicitly designate the identity and input tape of the receiving ITI. Each experiment comprises two special ITIs: The environment $Z$, and the adversary $\mathcal{A}$ (in the real experiment) or the simulator $S$ (in the ideal experiment). The environment $Z$ is the ITI that is initially activated. If any ITI completes its activation without giving any output, the environment is activated again as a fallback. If the environment $Z$ provides subroutine output, the whole experiments stops. The output of the experiment is the output of $Z$. Without loss of generality, we assume that $Z$ outputs a single bit only.

The Control Function and Message Delivery

If an ITI writes a message onto its outgoing message tape and calls external write, a control function decides if the operation is allowed\(^{10}\). If so, the experiment proceed as follows: If the receiver is uncorrupted and the designated input tape is either input or subroutine output, the message is copied to the respective tape of receiver. Else (meaning if the message is intended to be sent to an incoming message tape or the receiver is corrupted) the message is delivered to the respective tape of the adversary. This captures the natural intuition that input and subroutine output normally occurs within the same physical party and thus should be authenticated, immediate, confidential and of integrity. In contrast, external communication is only possible through an unreliable network under adversarial control.

UC Framework Conventions

In the UC framework, many important aspects are unspecified. For example, it leaves open which ITI is allowed to invoke what kind of new ITIs. The conventions stated in the following are probably the mostly used ones and quite natural.

Each party is identified by its party identifier (PID) $\text{pid}$ which is unique to the party and is the UC equivalent of the physical identity of this party. A party runs a protocol $\pi$ by means of an ITI which is called the main party of this instance of $\pi$. An ITI can invoke subsidiary ITIs to execute sub-protocols. A subsidiary and its parent use their input/subroutine output tape to communicate with each other. The set of ITIs taking part in the same protocol but for different parties communicate through their incoming message tapes. An instance

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\(^9\) Beware: Despite its name, this tape is actually an input tape as it receives subroutine output.

\(^{10}\) N.b.: The control function is another ITI that exists “outside” of the experiment and checks which combination of sender ID, receiver ID and message tape are feasible. For example, only subsidiary ITIs are typically allowed to provide subroutine output to their main ITI. For details see [11].
of a protocol is identified by its session identifier (SID) \( \text{sid} \). All ITIs taking part in the same protocol instance share the same SID. A specific ITI is identified by its ID \( \text{id} = (\text{pid}, \text{sid}) \).

The (Dummy) Adversary
The adversary \( \mathcal{A} \) is instructed by \( \mathcal{Z} \) and represents \( \mathcal{Z} \)’s interface to the network. To this end, all messages from any party to a party that has a different main party and that are intended to be written to an *incoming message* tape are copied to the adversary. The adversary can process the message arbitrarily. The adversary may decide to deliver the message (by writing the message on its own outgoing message tape), postpone or completely suppress the message, inject new messages or alter messages in any way including the recipient and/or alleged sender.

\( \mathcal{Z} \) may also instruct \( \mathcal{A} \) to corrupt a party. In this case, \( \mathcal{A} \) takes over the position of the corrupted party, reports its internal state to \( \mathcal{Z} \) and from then on may arbitrarily deviate from the protocol in the name of the corrupted party as requested by \( \mathcal{Z} \). This means whenever the corrupted ITI would have been activated (even due to subroutine output), the adversary gets activated with the same input.

A special case for the adversary is the so-called *dummy adversary* that reports all received messages to the environment and delivers all messages coming from the environment. It can be shown that the dummy adversary is *complete*, i.e. that if a simulator for the dummy adversary exists, then there also exists a simulator for any other adversary.

Ideal Functionalities and the Ideal Protocol
An ideal functionality \( \mathcal{F} \) is a special type of ITM whose instantiations (ITIs) bear a SID but no PID. Hence, it is an exception to the aforementioned identification scheme. Input to and subroutine output from \( \mathcal{F} \) is performed through dummy parties. Dummy parties merely forward their input to the input tape of \( \mathcal{F} \) and subroutine output from \( \mathcal{F} \) to their own outgoing message tape. They share the same SID as \( \mathcal{F} \), but additionally have individual party identifiers (PIDs) as if they were the actual main parties of a (real) protocol. The ideal functionality \( \mathcal{F} \) is simultaneously a subroutine for each dummy party and conducts the prescribed task. IDEAL(\( \mathcal{F} \)) is called the *ideal* protocol for \( \mathcal{F} \) and denotes the set of \( \mathcal{F} \) together with its dummy parties.

The UC Experiment
Let \( \pi \) be a protocol, \( \mathcal{Z} \) an environment and \( \mathcal{A} \) an adversary. The UC experiment, denoted by \( \text{Exec}_{\pi, \mathcal{A}, \mathcal{Z}}(n, a) \), initially activates the environment \( \mathcal{Z} \) with security parameter \( 1^n \) and input \( a \in \{0,1\}^* \). The first ITI that is invoked by \( \mathcal{Z} \) is the adversary \( \mathcal{A} \). All other parties invoked by \( \mathcal{Z} \) are set to be main parties of the challenge protocol \( \pi \). \( \mathcal{Z} \) freely chooses their input, their PIDs and the SID of the challenge protocol. The experiment is executed as outlined above.

Definition of Security
Let \( \pi, \phi \) be protocols. \( \pi \) *emulates* \( \phi \) in the UC framework, denoted by \( \pi \geq \phi \), if for every PPT adversary \( \mathcal{A} \) there is a PPT adversary \( \mathcal{S} \) such that for every PPT environment \( \mathcal{Z} \) there is a negligible function \( \text{negl} \) such that for all \( n \in \mathbb{N}, a \in \{0,1\}^* \) it holds that

\[
\left| \Pr[\text{Exec}(\pi, \mathcal{A}, \mathcal{Z})(n, a) = 1] - \Pr[\text{Exec}(\phi, \mathcal{S}, \mathcal{Z})(n, a) = 1] \right| \leq \text{negl}(n),
\]

where \( \text{Exec}(\pi, \mathcal{A}, \mathcal{Z})(n, a) \) denotes the random variable for the environment \( \mathcal{Z} \)’s output in the UC execution experiment with protocol \( \pi \) and adversary \( \mathcal{A} \) on input \( a \) and security parameter \( n \).

The simulator \( \mathcal{S} \) mimics the adversarial behavior to the environment as if this were the real experiment with real parties carrying out the real protocol with real \( \pi \)-messages. Moreover, \( \mathcal{S} \) must come up with a convincing internal state upon corrupted parties, consistent with the simulated protocol execution up to this point (dummy parties do not have an internal state).

Protocol Composition
UC security is closed under protocol composition: Let \( \pi, \phi, \rho \) be protocols. Then,

\[
\pi \geq_{\text{UC}} \phi \implies \rho^\pi \geq_{\text{UC}} \rho^\phi
\]

C.1 Bulletin Board Functionality
In our constructions, we make use of the ideal functionality \( \mathcal{F}_{\text{reg}} \) that models a public bulletin board.

Definition 6 \( \mathcal{F}_{\text{reg}} \).
\( \mathcal{F}_{\text{reg}} \) proceeds as follows:

- **Report:** Upon receiving a message \((\text{register}, \text{sid}, v)\) from party \( P \), send \((\text{registered}, \text{sid}, P, v)\) to the adversary; upon confirmation, record the pair \((P, v)\). Otherwise, ignore the message.
- Retrieve: Upon receiving a message $(\text{retrieve}, \text{sid}, P_i)$ from some party $P_j$ (or the adversary $A$), generate a public delayed output $(\text{retrieve}, \text{sid}, P_i, v)$ to $P_j$, where $v = \perp$ if no record $(P, v)$ exists.

Note that in contrast to the usual definition, we allow key revocation in $F_{\text{reg}}$. 