Unlinkable Policy-Compliant Signatures for Compliant and Decentralized Anonymous Payments

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ABSTRACT

Privacy-preserving payment systems face the difficult task of balancing privacy and accountability: on one hand, users should be able to transact privately and anonymously, on the other hand, no illegal activities should be tolerated. The challenging question of finding the right balance lies at the core of the research on *accountable privacy* that stipulates the use of cryptographic techniques for policy enforcement. Current state-of-the-art systems are only able to enforce rather limited policies, such as spending or transaction limits, or assertions about single participants, but are unable to enforce more complex policies that for example jointly evaluate both, the private credentials of sender and recipient, situations that occur in cross-border payments, let alone to do this without auditors in the loop during payment. This severely limits the cases where decentralized virtual assets can be used in accordance with regulatory compliance such as the Financial Action Task Force (FATF) travel rule, while retaining strong privacy features.

We present *unlinkable Policy-Compliant Signatures (ul-PCS)*, an enhanced cryptographic primitive extending the work of Badertscher et al. (TCC 21). We give rigorous definitions, formally proven constructions, and benchmarks using our prototype developed using CharmCrypto which gives the first insights into feasibility of PCS. Unlinkable PCS has the following unique combination of features: ① It is an enhanced signature scheme where the public key encodes in a privacy-preserving way the user's verifiable credentials (obtained from a credential authority). ② Signatures can be created (and later publicly verified) by additionally specifying a recipient's public key aside of the to-be-signed message. A valid signature can only ever be created if the attributes x_S of the signer and the attributes x_R of the receiver fulfill some global policy $F(x_S, x_R)$. ③ The signature can be created by the signer just knowing the recipient's public key; there is no further interaction needed and no information is leaked (beyond the validity of the policy). ④ Once credentials are obtained, a user can generate fresh public keys without interacting with the credential authority.

By merging the act of signing a transaction with the act of providing an assurance about the involved participants being compliant with complex policies, yet retain that participants are able to change public keys without the involvement of an authority, we formally show how ul-PCS is a step towards improving regulatory compliance of privacy coins such as Monero or Zcash.

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Digital Signatures, Decentralized Anonymous Payments, Accountable Privacy, Policy-Compliant Signatures

1 INTRODUCTION

The inception of Bitcoin [55] and its novel approach to implement a transaction ledger via a blockchain brought to light a new type of payment system that does not rely on trusted parties like a central bank, but instead uses distributed ledger technology to settle direct transactions between parties and to protect against double-spends. Besides Bitcoin, decentralized anonymous payment (DAP) systems, such as Zcash [12] and Monero [3], have been proposed to improve privacy and anonymity guarantees. In these systems, parties enjoy full transaction privacy and anonymity, which makes it challenging to hold parties accountable for their actions, let alone for a regulator to be assured regulatory compliance is met. This led to the study of accountability and auditability in the context of distributed payment systems with the main goal of understanding the necessary adoption requirements of these systems in various jurisdictions [26].

The core meaning of auditability is to provide means to a regulator to ask for specific pieces of information, based on a legal system defining a catalogue of admissible questions, and be given the answer in a faithful way [26]. Clearly, an auditor only needs to (reactively) ask for information that the system does not proactively enforce by itself. This policy enforcement is precisely how accountability for private payment systems, or *accountable privacy* for short, has been defined in [26, 42]. However, in many of the currently proposed systems, the granularity of accountable privacy is not very high, and the focus lies on (functions about) the monetary value of transactions or spending limits on accounts [25, 42, 56, 64], where more centralized designs typically allow for a richer set of provable statements. To make matters worse, auditability is often equated explicitly or implicitly with the ability of an auditor to revoke the privacy and anonymity of transactions of any individual user (or given a key to supervise or view the transaction log) [4, 9, 23, 28, 30, 52]. While this trivially avoids the need for more sophisticated policy enforcement techniques, it goes without saying that such a powerful revocation capability is problematic as it could be subject to abuse. In light of this, an important question arises:

How fine-grained can we enforce policies on the transaction level?

A blueprint followed by several works [35, 61, 65] to obtain accountable privacy in DAP systems is to consider transaction types for specific use cases where each use case is governed by a policy whose compliance can be enforced. In exchange for a potentially

limited scope, users get back full and unrevocable anonymity for these transactions. For example the UTT system [61] defines a digital analogon of cash: the so-called budget coins are issued in a limited fashion to certified users. As soon as the user has obtained the coins, they can transact in a cryptographically secure way that, among other things, ensures full, unrevocable privacy.

Central to the success of such systems is the level of granularity for which one can enforce policy compliance. The richer the class of cryptographically enforceable policies, the easier it is to define different transaction types. In view of the developing ecosystem on digital credential systems [20, 27, 63], more legal policies can be translated to the digital world, such as predicates about which two individuals are allowed to transact based on age or residency, or on accredited attributes like financial reliability, credit worthiness, or other real-word certifications. More concretely, this leads to more fine-grained transaction types for which full privacy can be guaranteed. An example could be to lift certain limits for, e.g., residents spending coins in shops that are certified to only sell goods for everyday life. If such policies are enforceable by cryptographic means in a DAP system, this would heavily boost the privacy of users while satisfying regulatory needs. Unfortunately, there is no DAP system that supports this functionality and integrates smoothly with a credential issuance infrastructure. One reason is the strong, at first sight contradictory, set of requirements, which are that (1) all credentials of a user must remain private, even during payment; (2) it must be possible to perform policy evaluation jointly on both, sender and receiver, attributes (such as whether they have the same residence, or whether they belong to jurisdictions across which money transfers are permitted); and (3) compatibility with DAP systems must be guaranteed, which means that evaluation must be possible whenever the sender knows just the recipient's public key (and no further interaction is required to submit the transaction) and that compliance is publicly verifiable. We note in passing that even centralized designs such as Platypus [65] currently do not support this stronger type of joint policy evaluation (but admit individual attestations of users about their own attributes toward the central bank).

In this work, we give a generically applicable cryptographic policy-enforcement mechanism that is compatible to be sued in tandem with a DAP systems, i.e., it satisfies properties (1)-(3) above. The mechanism is generic—it has the interface of a signature scheme—and can be composed with larger systems to complement existing solutions to achieve fine-grained accountable privacy, either via use in smart contracts, as a layer-2 system in connection to a mainchain, or updating the address format of a mainchain (via a hard fork). However, our solution is not limited to decentralized ledgers, and can also be applied to centralized designs to reduce the information leakage about a transaction towards the auditor.

1.1 Contributions

Definitions. We introduce the enhanced cryptographic primitive called unlinkable policy-compliant signatures, a stronger version of policy-compliant signatures introduced by Badertscher, Matt, and Waldner [7]. We give precise definitions of unforgeability, attribute-hiding, and unlinkability. Since privacy (resp. anonymity) and unforgeability are intertwined in this definition, special care must be taken to arrive at a reasonable definition.

Generic solution. We provide a generic solution to the problem that realizes ul-PCS for arbitrary policies $F(x, y)$ defined for sender and receiver attributes x and y, respectively, and formally prove its security. Despite its seemingly theoretical focus, the main practical challenges in instantiating this primitive are the predicate encryption (PE) scheme and the non-interactive zero-knowledge (NIZK) proof systems. We present an implementation of our generic construction for the inner-product (IP) predicate, i.e., for vectors x and y of attributes (encoded as field elements) such that $F(x, y) = 1$ iff the inner product of x and y is zero. This predicate is known to be sufficient to realize many real-world policies including set membership (e.g., used in identity-based revocation systems), CNF formulas and exact threshold clauses (with conjunctive or disjunctive clauses) as well as hidden-vector encryption enabling various sorts of comparisons as well as conjunctions of the above statements [7].

More efficient constructions and implementations. We provide two additional, specific constructions for certain policy classes that are more lightweight in terms of required cryptographic tools and have the additional features of constant-sized public keys and signatures, as well as constant verification time. We provide (academic) prototype implementations in Python and utilize the Charm-crypto framework [2] for all constructions we present in this work.

For the experiments, we use a PC (laptop) with Intel Core i7-9850H CPU @ 2.60 GHz and 16 GB memory over the BN-254 curves. Our benchmarks demonstrate that the signing execution time for the specific schemes is less than 2 seconds for reasonably complex policy sizes. The verification procedure of signatures takes around 1.6 seconds, with public keys in the order of 28 kbytes, and signature sizes of about 16 kbytes (all independent of the number of attributes). Our prototypes for the specific policy classes thus suggest that, although ul-PCS may at first sight appear like a heavy cryptographic tool, they can enforce policies with reasonable practical efficiency.

Finally, our prototype for the generic solution gives a first estimate about the real-world complexity of general-purpose PCS designs. Due to its relationship with predicate-encryption (which we explain in Section 2 below), the performance is largely influenced by the choice of PE scheme. We run our benchmarks in the range of $n = 5$ up to 50 attributes. Signing takes 3 seconds for $n = 5$ and each additional attribute incurs, on average in that range, an estimated cost of 340 ms. For verification, we obtain around 4.5 seconds for $n = 5$ with an average cost per additional attribute of around 420 ms. Public key sizes on the other hand grow linearly, starting at 79 kbytes for $n = 5$, and incurring a cost of roughly 9.9 kbytes per additional attribute (which means that even for 50 attributes, we have key sizes similar to McEliece cryptosystems). Signatures are about 42 kbytes for $n = 5$ and grow by 5.14 kbytes per additional attribute. We compare these characteristics with the suggested PCS construction from [7], for which we provide the first prototype (using the same underlying PE scheme), which enables us to directly observe the overhead that our generic anonymity enhancement techniques impose.

Application to payment systems and beyond. We formally show how to combine ul-PCS with UTxO ledgers, as well as with DAP systems like Zcash or Monero to ensure fine-grained policies on the transaction level without the involvement of an auditor. To this aim, we build on a recent abstract framework by Engelmann et al. [33] (PETS'22) to modularly compose ul-PCS with so-called one-time accounts, effectively coupling addresses with private credentials. This proposal therefore puts forth an enriched Layer-1 address format (and thus would either require a hard fork or would have to be a sidechain with this added functionality). We point out that this solution does not introduce any additional trust assumptions beyond what a credential issuance infrastructure would need. We further show that credential issuance, more formally, setup and key-generation, can be distributed across a set of servers to avoid a single point of failure. This is an important consideration, since corrupting the credential issuer usually enables an adversary to generate keys by itself, which it can then use to brute force the attributes of every participant in the system by checking to which participants it is able to send based on its self-issued attributes. Finally, in the centralized settings of CBDC constructions [52, 65], we showcase how the sue of ul-PCS extends the set of policies that could be automatically enforced.

2 TECHNICAL OVERVIEW AND RELATED WORK

In this section, we give an in-depth overview of the technical contributions of this work. We describe the security goal of ul-PCS and the inherent complications to realize such a strong notion. Afterwards, we outline the constructions we present in this work and the different types of policies that we support. Finally, we provide an overview how DAPs can be enhanced with ul-PCS in the context of FATF regulations [37], finding a good balance between privacy and regulatory-friendliness, as outlined in the previous section. We conclude with an overview of related work.

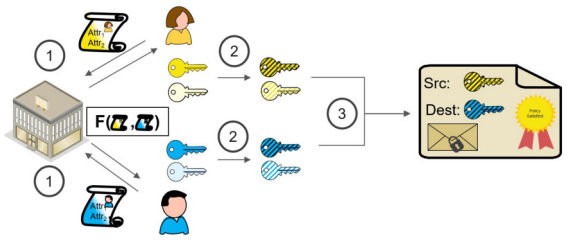


Figure 1: Usage of an unlinkable policy-compliant signature scheme: 1.) Alice with credentials x and Bob with credentials y, run through a registration process with a credential authority. 2.) At any point, they can decide to re-randomize their keys in order to break any link to their previous actions. 3.) Alice generates a signature, e.g., on a private transaction, with Bob's public key as the destination address. The fact that a valid signature emerges proves that $F(x, y) = 1$.

2.1 Realizing ul-PCS for Various Policy Types

We begin by recalling the model behind PCS, put forth in [7]. The model involves three main roles as depicted in Figure 1: the Credential Issuing Authority (CA), Signers and Verifiers. The policy can be defined for a set of senders' attributes $\mathcal{S}$ and receivers' attributes $\mathcal{R}$ such that a predicate function $F : \mathcal{S} \times \mathcal{R} \to \{0, 1\}$ determines which senders, with a given set of attributes, are allowed to create a signature for which receivers, again with a set of attributes. If x and y denote the (private) attributes of sender and receiver, respectively, then creating a valid signature is allowed iff $F(x, y) = 1$. *Existential unforgeability* demands that a signer cannot generate a valid signature for a recipient, identified by its public key (presumably encoding attributes y), unless it has obtained the secret key associated with x (issued by the CA) such that $F(x, y) = 1$. *Attribute hiding* guarantees that nobody learns any meaningful information about the attributes of the signer and targeted verifier except, of course, the bit that they jointly fulfill the policy when a valid signature emerges. We introduce a missing feature: *unlinkability*. A user must be able to change the representation (i.e., its public key) without interacting with the authority, to break the link between its actions—while retaining all security features above. Since ul-PCS combines anonymity with, for example, unforgeability requirements, the existing security games must be adapted. A resulting challenge is that winning conditions must remain well-defined, even if keys are re-randomized (possibly done by the adversary) and attributes are hidden.

Design challenges. At first sight, the problem might appear not too difficult as it (superficially) resembles anonymous credential (AC) systems [21, 27], which are well-studied and play a key role in privacy-preserving applications by enabling users to authenticate themselves while ensuring the unlinkability of their actions. A credential in this context is typically a signature on the attributes. During the authentication process, users can demonstrate their possession of a credential that satisfies a specific access policy without revealing any details about the real identity of the user, except that they meet the criteria of the access policy. While one can notice some similarities with our goal set out above, ul-PCS possesses distinctive properties that deviate crucially from the intuition about how AC systems are used.

The first difference is in the representation of the credentials. In ul-PCS the credential is an inherent part of the public information as its intended use case is as an address in a payment system, cf. Section 2.2. In contrast, credential systems typically assume that claims do stay private until shown in a credential-show operation. In many implementations, a credential is simply a set of attributes signed by an authority. Our re-randomization requirement means that we have to find a privacy-preserving representation of users' credentials that is fully re-randomizable without contacting the authority again. This departs from the standard requirements of an AC system, which mainly focus unlinkability between two credential-show operations toward a service provider.

Furthermore, generic credential-show algorithms are not necessarily privacy-preserving, unless they involve assertions proved in zero-knowledge. In this context, these are properties that a user proves about its private attributes. In contrast, in order to fulfill the desired goals of ul-PCS the signer/sender needs to assert a

policy that involves the private attributes of itself and the receiver—without having the receiver disclose the information to the sender. Thus, any ul-PCS system must satisfy a set of seemingly incompatible requirements which makes the problem highly non-trivial to solve.

Scheme for generic policies. We first consider the case for generic policies that achieve the goal of an arbitrary joint check of the sender's and receiver's attributes encoded (in a privacy-preserving manner) in their public keys. The standard PCS construction in [7] relies on three main cryptographic primitives: (Predicate-Only) Predicate Encryption, Digital Signatures and Non-Interactive Zero-Knowledge (NIZK) proofs which can all be instantiated in the standard model. The high-level idea of this construction is that public keys of parties, acting as receivers, contain a PE-ciphertext decryptable by the signer/sender, only upon policy satisfaction. The signature and NIZK are needed to achieve unforgeability and to prove multiple relations during the signing process. Following this paradigm, we present the first unlinkable PCS scheme supporting any policy in Section 4. We build our scheme using those building blocks and integrate a method that allows a party to evolve its public key, according to a pseudo-random sequence tied to their attributes. A critical hurdle to overcome in this setting is that, during the process of refreshing the key, a party cannot add new attributes that have not been issued to that party. We present more details on this in the technical sections.

2.1.1 Schemes for more specific policy classes. Despite the fact that PE is the most elegant conceptual fit for general PCS, it impacts efficiency and public-key sizes since there is a direct implication between PCS and PE. In more detail, the reduction presented in [7] shows that PCS gives rise to PE encryption (for a related predicate), and, furthermore, that a PCS public key can be turned into a PE ciphertext. This implication becomes even more dominant in the ul-PCS case since, in this case, it is required, as part of the construction, to prove the well-formedness of the public key. To improve this situation, we show how, for specialized policies, it is possible to avoid PE in order to obtain more practically performing schemes. We consider two specific policy classes:

Scheme for role-based policies. We consider role-based access-control (RBAC) matrices. Such a matrix can be seen as a function $F : [n_R] \times [n_R] \to \{0, 1\}$ (for a given, presumably relatively small, set of n_R number of roles) and captures which roles i can transact towards which role j, namely iff $F(i, j) = 1$. Depending on the structure of such a matrix, one can implement a wide range of access control policies, where "access control" here rather means which role is allowed to send a signed message (or transaction) to which other role akin to information flow in [22]. Of particular interest is the special case of equality, i.e., the identity matrix $F(i, j) = 1$ iff $i = j$ [5] as we recall below. We present an approach based on accumulators (which are realizable based on pairing-based signatures of a specific type) instead of PE. For general RBAC matrices, the scheme satisfies what we call outsider-secure attribute hiding, sometimes referred to as transaction-graph obfuscation or confidentiality [23, 26] (aside of unforgeability and unlinkability). This security notion captures the inability of an attacker to infer any useful information by just analyzing the transaction graph of a blockchain. For the special case of the equality policy the scheme satisfies all security properties (in particular full attribute-hiding). The equality policy is particularly useful in contexts where users and/or services are clustered into groups or categories based on their real-world credentials, and to ensure that transactions are conducted within those groups.

Scheme for separable policies. Separable policies are policies that admit the simple representation $F(x, y) = S(x) \wedge R(y)$, and thus belong to the important class of predicates w.r.t. individual assertions about an entity's attributes, e.g., the ones supported by centralized solutions like Platypus [65]. We show that those policies can be realized by unlinkable PCS schemes in an efficient way, where the PE part can be replaced by standard public-key encryption. We point out some of the applications of these policies: on one hand, $S(x)$ could be the predicate that a sender has undergone KYC regulations, while a priori anyone can be a receiver ($R(y) = 1$). Translated to our scheme, and its associated usage in a DAP system, this means that anyone can immediately start off and receive coins, while only being able to spend them later, once KYC regulations have been met. The second, more technical use-case, appears in Zcash-like DAP systems: the three transaction types in Zcash, namely Mint, Burn, and Pour transactions can directly be captured as simple attributes, such that Mint is an action from a sender-only public key / address ($S(x) = 1, R(x) = 0$, no receiving possible), Burn is a transaction toward a receiver-only public key ($S(x) = 0, R(x) = 1$, no sending possible), and the normal use-case is a user that can send and receive ($S(x) = R(x) = 1$). Combining this observation with the results of Section 6, gives a generic way to let the monetary policy be governed by accredited users while preserving their privacy and anonymity. In addition, when integrating Zcash with other ledger-based currencies, one can steer which users are allowed to convert base currency in exchange for newly minted zerocoins.

2.2 DAPs, FATF, and ul-PCS

Virtual assets is the technical and legal term referring to decentralized digital tokens that are considered cryptocurrencies. Such virtual assets can either be stored in self-hosted wallets, or stored in a hosted (or custodial) wallet on a virtual-asset exchange, more generally referred to as virtual-asset service provider (VASP) [37]. While digital assets serve real financial and investment needs, to protect the ecosystem from fraudulent and criminal activities, the Financial Action Task Force (FATF) demands that VASPs comply with the so-called travel rule[1]. The travel rule mandates that VASPs maintain identifying information behind any address they store, as well as to collect and exchange information about sender and receiver when funds move from one hosted wallet to another, and further verify that certain (legal) policies are satisfied, such as restrictions on capital flow between jurisdictions of the involved entities.

The travel rule puts a lot of burden on VASPs. Identifying the financial beneficiary behind any address is similar to solving the lookup problem in PKI infrastructures: it must be efficiently possible for any VASP, when given an address addr, to obtain the identifying information behind addr, and most importantly, the VASP

[1] https://sanctionscanner.com/blog/financial-action-task-force-fatf-travel-rule-140

that is hosting addr (if any). Since these checks are the precursor for sensitive information exchanges about financial individuals, the accuracy of such an association is of utmost importance: a (curious) VASP should not be able to learn such information if it cannot present a proof that it is the custodian of either the sending or receiving wallet. It must further also be possible to determine whether a wallet is hosted at all. Realizing such a lookup service as an overlay over decentralized tokens is a difficult endeavor, as personal information is stored, maintained, transferred, and replicated on various VASPs, which is not just a concern related to privacy, but also mandates that information about an individual must be consistent. Even if these issues were resolved, it appears that the FATF travel rule does not align well with anonymous payment systems. This is due to the strong anonymity guarantees that these assets offer, which deems them suspicious, mainly due to a lack of technological capabilities of reconciling accountability and privacy efficiently for decentralized assets.

In this paper, we put forth a mechanism, which we formalize later in Section 6, enabling the reconciliation of the above views, the silver lining being that an address addr already provides an encoding of credentials with it—encoded in a privacy-preserving way via the help of a credential authority issuing any sort of attributes to participants. This achieves a separation of duties: the identifying information is carried by the address itself, and its privacy features, such as recoverability or privacy revocation features, is up to the credential system, not the VASPs. While the idea of connecting addresses with credentials is not really novel, what ul-PCS adds to the system is the combination of two new features:

1. It enables that an address carries anonymous credentials, but also has the look-and-feel of common cryptocurrencies: a user can create fresh addresses by itself that carry the same information, without the need to contact the credential authority.

2. Asset transfers can be automatically governed by a policy $F(x, y)$, where x are the attributes of the sender and y are the attributes of the receiver. That is, a transaction transferring a token from addr_x to addr_y can never be accompanied by a valid signature unless $F(x, y) = 1$. Furthermore, nothing more is leaked by such signatures other than the validity of the policy.

These two features combined can improve the complex situation faced by VASPs considerably, while maintaining user privacy. As we define formally in Section 6.1, compliance checks based on ul-PCS and spending rights in a DAP system can be seen as two separate steps, similar to multi-signatures. A VASP can formally host an address by controlling just the DAP private key, while the ul-PCS private key always remains with the user. This separation further limits the impact of a (potentially misbehaving) credential issuer in the transaction system. To conduct an asset transfer, the user and the VASP must both provide the signature. Still, a user can have many different unlinkable addresses with various VASPs thanks to the re-randomization property that allows it to create fresh addresses. Finally, while the above solution works best if the underlying blockchain allows native support of such addresses and multi-signatures, we point out that blockchains with smart-contract capabilities can support these types of operations by defining a custom token controlled by two keys.

Asset transfers implemented this way are guaranteed to follow the policy—without ever requiring from the involved parties to disclose any information about their attributes–and reliefs the VASPs from collecting (or transmitting) information that are made for the sole purpose of checking compliance of the mentioned policy (of course, there might still be a need to collect information that is not formalized by a digital policy in which case ul-PCS helps reducing the amount of collected information). Furthermore, the asset transfer is non-interactive in the same sense as common cryptocurrencies are: the sender does not need more information to transfer the asset than the knowledge of the recipient's address.

It further allows the VASP to outsource the task of KYC to accredited authorities. Here, the authorities issue attributes to reflect a user's KYC status which are then in charge of delivering the associated information, if requested by legal enforcement. We discuss such possibilities in Section 6.4. Furthermore, a VASP itself (such as a mixing service) can carry a ul-PCS key representing attributes accredited to it. This way, a policy can specify what types of users are allowed to use which specific services. Only those users would be able to transfer assets into an account of a VASP that satisfies the rules. We discuss this example in more detail in Section 6.3.

In summary, the proposed approach is a paradigm change in handling accountability in transaction systems. Instead of enforcing an overlay-structure where every VASP collects the same type of information, which arguably is rather intrusive, we put forth a cryptographic mechanism that turns this view upside down. More precisely, we intimately connect the addresses to the credentials with automated compliance checks, while allowing a user to be represented by fresh addresses with different services.

2.3 Prototypes and Benchmarks

We provide a full prototype implementation for all the schemes we present in this work. These are the first working prototypes of policy-compliant signatures in general, for which we provide the results in Section 5. Our prototype should be seen as an academic prototype that contains a first faithful implementation of all building blocks including the zero-knowledge proof systems, however, without production-grade optimizations (we mention a few as open directions). Yet, even without these modifications, the main benefit of the prototypes, besides obtaining concrete runtime estimates, is the required dovetailing of the zero-knowledge system with other cryptographic primitives. It is highly non-trivial how a concrete instantiation of our generic scheme would actually look like (and to what extent it follows from standard tools). For that instantiation, we pick a predicate-encryption scheme for the class of inner-product (IP) policies [57]. Those schemes are theoretically efficient and inner-product predicates are known to realize various complex policies [51] such as DNF/CNF formulas, threshold clauses, or polynomial evaluations, which directly translate to the PCS setting [7]. Furthermore, since hidden-vector encryption can be realized from IP, it follows that IP is sufficient to implement all policies from [17].

The main challenge to overcome from a practical perspective is to achieve that a user is able to provably re-encrypt the particular PE ciphertexts without introducing new attributes. While this is easy on paper, we must implement this securely by using a combination

of structure-preserving signatures on equivalence classes and the observation that the particular PE ciphertexts are closely related to generalized Pedersen commitments for which we can achieve re-randomization via its homomorphic property. We thereby are able to show that we can couple the particular PE scheme, which is based on dual pairing vector spaces, with standard NIZK systems (such as Groth-Sahai and Sigma protocols). The full specification of all our prototypes are given in Appendices F and G.

2.4 Related Work

Since ul-PCS are signatures, they can be composed with any transaction system to capture more fine-grained policies. We already contrasted this paper with [7], which serves as the basis we extend. Therefore, we now focus on an overview of how ul-PCS can improve the expressiveness of existing payment systems. We present the technical details on this later in Section 6.

In the context of distributed payment systems, the focus of prior works that support accountable privacy is on using NIZKs to prove statements about the content of a transaction such as, e.g., a sequence of transactions are below a spending limit in total or below a receiving limit in total [42, 61, 64]. These policies are extremely useful and the involved NIZKs are practical. The systems are therefore able to publicly convince an auditor that certain actions are within limits but do not give assertions about the credentials of the involved parties and it seems hard to obtain unrevocable privacy for more than cash-like transactions [61]. Enriched with digital credentials, however, more classes of transactions can be defined. Parties involved in a payment could be accredited (trusted) institutions or shops, for which a sending or receiving limit is lifted. A PCS signature signing the transaction can assure money flow only between two such institutions. Furthermore, certain coins can be issued for a specific purpose to individuals, or capital flow control can be ensured based on the credentials tied to a person's public key, and the PCS signature can attest compliance.

In the context of recent CBDC proposals [52, 65], Platypus [65] is a very elaborate and nuanced system. During payments, where interaction with a central bank is required, it is proposed to carefully distinguish types of transactions and, depending on this, the bank might request further information, in plain or via zero-knowledge proofs. Although being centralized, the system does not admit to prove statements about sender and receiver of a transaction simultaneously, e.g., to control whether cash flow between two individuals is compliant with capital control. In such a scenario, information needs to be revealed to the central bank. However, the approach we take to make this possible in ul-PCS can be directly applied to such centralized designs and enrich them with even more fine-grained policies. The same holds for Peredi [52], which, compared to Platypus, does not put forth a fine-grained transaction model and presents a revocation-based auditability solution.

Finally, in a recent work on updatable PCS [6] it is shown how to update the policy without changing the keys of the users. In this setting, an interactive version of PCS is considered with support by generic two-party protocols (2PC), but where public keys cannot be re-randomized. Our approach of adding unlinkability would be applicable to that setting too and our runtime estimates can serve as a lower bound for their schemes due to their higher complexity.

3 UNLINKABLE PCS: FORMAL DEFINITION

Now, we present the syntax of unlinkable policy-compliant signatures (ul-PCS) that match the outlined use cases above. We discuss ways to decentralize the setup and key generation later in Section 6. In the technical sections that follow, we follow standard notation which we recall in Appendix A for completeness.

Definition 3.1 (Unlinkable Policy-Compliant Signatures). Let $\{X_\lambda\}_{\lambda\in\mathbb{N}}$ be a family of attribute sets and denote by $\mathcal{X}_\lambda$ the powerset of X_λ. Further let $\mathcal{F} = \{\mathcal{F}_\lambda\}_{\lambda\in\mathbb{N}}$ be a family of sets $\mathcal{F}_\lambda$ of predicates $F\colon \mathcal{X}_\lambda \times \mathcal{X}_\lambda \to \{0,1\}$. Then an *unlinkable policy-compliant signature (ul-PCS)* scheme for the functionality class $\mathcal{F}_\lambda$ is a tuple of five PPT algorithms ULPCS = (Setup, KeyGen, RandKey, Sign, Verify):

$\mathsf{Setup}(1^\lambda, F)$**:** On input a unary representation of the security parameter λ and a policy $F \in \mathcal{F}_\lambda$, output a master public and secret key pair $(\mathsf{mpk}, \mathsf{msk})$.

$\mathsf{KeyGen}(\mathsf{msk}, x)$**:** On input the master secret key msk and a set of attributes $x \in \mathcal{X}_\lambda$, output a public and secret key pair $(\mathsf{pk}, \mathsf{sk})$.

$\mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})$**:** On input the master public key mpk and a secret key sk, output a new public-secret-key pair $(\mathsf{pk}', \mathsf{sk}')$.

$\mathsf{Sign}(\mathsf{mpk}, \mathsf{sk}_S, \mathsf{pk}_R, m)$**:** On input the master public key mpk, a sender secret key sk_S, a receiver public key pk_R and a message m, output either a signature σ or $\perp$.

$\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}_S, \mathsf{pk}_R, m, \sigma)$**:** On input the master public key mpk, a sender public key pk_S, a receiver public key pk_R, a message m and a signature σ, output either 0 or 1.

Correctness. A ul-PCS scheme is correct if in any execution, honestly generated signatures computed using honestly generated private and public keys, potentially re-randomized multiple times, reflect the policy. Compared to standard PCS, it is easier to capture this as a correctness experiment, since the interaction introduced with re-randomization leads to more complex scenarios. We present the formal game in Appendix B.1.

Detectability Relation. Compared to the requirements of a standard PCS scheme, the requirements for an unlinkable PCS scheme pose a definitional challenge: we need to capture unforgeability and policy-compliance in a security game but, at the same time, keys are randomized (potentially by the adversary) and no efficient algorithm could detect whether this is in fact a valid forgery—since all attributes are private and parties are not traceable. We solve this definitional issue by introducing what appears to be a quite natural requirement: any ul-PCS scheme must satisfy a detectability relation which intuitively captures the property that a party, knowing its own initial secret key, can detect whether a valid public key is in fact derived from it (that is, a party can detect its own public keys in a ledger). Using this detection property, the challenger in the security game can determine which keys belong to which oracle queries. The algorithm is called Detect, and takes as input a target public key, and the keys generated by the challenger for different parties. The algorithm must return the index of the party that the target key belongs to. Looking ahead, this must hold even if the adversary is in charge of re-randomizations. Clearly, such an algorithm must satisfy a non-triviality condition: when keys are honestly generated and re-randomized, the algorithm detects only

correct relations and never confuses parties.[2] The formal definition of detectability can be found in Appendix B.2. We point out that this form of detectability is very different from tracing—the property or feature that an additional entity, the auditor, is able to trace parties by means of a special viewing or revocation key. This property is not entirely new and has already been introduced in the context of, e.g., Monero [3]. In Section 6, we give more details on how to embed ul-PCS in larger contexts and discuss the traceability requirement appearing in the literature on CBDCs.

3.1 Security Games

Before presenting the notions of unforgeability, attribute-hiding and unlinkability, we describe the adversarial capabilities in the different security games. To keep track of all honestly generated keys, corrupted keys and generated signatures, we define the initially empty sets $\mathcal{QK}$, $\mathcal{QC}$ and $\mathcal{QS}$, respectively.

Key-Generation Oracle $\mathsf{QKeyGen}(\cdot)$**:** On the i-th input of an attribute set x_i, generate $(\mathsf{pk}_i^0, \mathsf{sk}_i^0) \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x_i)$, and add $((i,0), \mathsf{pk}_i^0, \mathsf{sk}_i^0, x_i)$ to $\mathcal{QK}$. Return pk_i^0.

Left-or-Right Key-Generation Oracle $\mathsf{QKeyGenLR}_\beta(\cdot,\cdot)$**:** On the i-th input of a pair of attribute sets $x_{i,0}$ and $x_{i,1}$, generate $(\mathsf{pk}_i^0, \mathsf{sk}_i^0) \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x_{i,\beta})$, add $((i,0), \mathsf{pk}_i^0, \mathsf{sk}_i^0, x_{i,0}, x_{i,1})$ to $\mathcal{QK}$, and return pk_i^0.

Key-Randomization Oracle $\mathsf{QRandKey}(\cdot)$**:** On input an index i, if $\mathcal{QK}$ contains entries $((i,0), \mathsf{pk}_i^0, \mathsf{sk}_i^0, \dots), \dots, ((i,c_i), \mathsf{pk}_i^{c_i}, \mathsf{sk}_i^{c_i}, \dots)$, then compute $(\mathsf{pk}_i^{c_i+1}, \mathsf{sk}_i^{c_i+1}) \leftarrow \mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk}_i^{c_i})$ and add $((i, c_i+1), \mathsf{pk}_i^{c_i+1}, \mathsf{sk}_i^{c_i+1}, \dots)$ to $\mathcal{QK}$ and return $\mathsf{pk}_i^{c_i+1}$.

Corruption Oracle $\mathsf{QCor}(\cdot)$**:** On input an index i, if $\mathcal{QK}$ contains entries $((i,j), \mathsf{pk}_i^j, \mathsf{sk}_i^j, \dots)$ for $0 \le j \le c_i$ for some $c_i \ge 0$, then copy these entries from $\mathcal{QK}$ to $\mathcal{QC}$ and return the list $(\mathsf{sk}_i^0, \dots, \mathsf{sk}_i^{c_i})$.

Signing Oracle $\mathsf{QSign}(\cdot,\cdot,\cdot)$**:** On input an index pair (i,j), a public key pk' and a message m, if $\mathcal{QK}$ contains an entry $((i,j), \mathsf{pk}_i^j, \mathsf{sk}_i^j, \dots)$, then compute $\sigma \leftarrow \mathsf{Sign}(\mathsf{mpk}, \mathsf{sk}_i^j, \mathsf{pk}', m)$, add $((i,j), \mathsf{pk}_i^j, \mathsf{pk}', m, \sigma)$ to $\mathcal{QS}$ and return the signature.

Randomization-Challenge Oracle $\mathsf{QRandKey}_\beta(\cdot)$**:** On receiving a query i, do the following: if $\beta = 0$ then set $(\mathsf{pk}', \mathsf{sk}') \leftarrow \mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})$, and if $\beta = 1$ set $(\mathsf{pk}', \mathsf{sk}') \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x)$, where x is taken from the entry $((i,0), \mathsf{pk}, \mathsf{sk}, x)$ of $\mathcal{QK}$, and sk is taken from the entry $((i,j), \mathsf{pk}, \mathsf{sk}, x)$ of $\mathcal{QK}$ with highest j for the given i. Add $((i, j+1), \mathsf{pk}', \mathsf{sk}')$ to $\mathcal{QK}$ and return pk'.

Notice that the randomization-challenge oracle is one way of formalizing key evolution. Following our application story, we assume a party updates its most recent key (similar to key-evolving signatures). Other equivalent options are possible as well. For notational convenience, if the set $\mathcal{QK}$ contains the sequence $((i,0), \mathsf{pk}_i^0, \mathsf{sk}_i^0, \dots), \dots, ((i,c_i), \mathsf{pk}_i^{c_i}, \mathsf{sk}_i^{c_i}, \dots)$ we denote by $\mathcal{QK}_i$ the sequence of keys $[(\mathsf{pk}_i^0, \mathsf{sk}_i^0), \dots, (\mathsf{pk}_i^{c_i}, \mathsf{sk}_i^{c_i})]$ of party i.

3.2 Security of ul-PCS

Unforgeability. Unforgeability captures the property that signatures by honest parties cannot be forged and that it is not possible to create valid signatures that are not policy-compliant. In more detail, an adversary $\mathcal{A}$ creates a valid forgery if: (a) it is able to generate a valid signature for a public key belonging to an honest/uncorrupted party, or (b) it is able to generate a valid signature for a key that has never been issued for an attribute set[3], or (c) it is able to generate a valid signature for a key pair $\mathsf{pk}_S, \mathsf{pk}_R$ where the corresponding attributes do not fulfill the policy F. We capture all these conditions in the security game in Figure 2. To efficiently evaluate condition (c), we make use of the mentioned detection algorithm since not all keys are generated by the challenger, but potentially modified keys by an adversary. The formal definition can be found in Appendix B.3.

$\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A})$

$(F, \mathsf{st}) \leftarrow \mathcal{A}_1(1^\lambda)$

$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, F)$

$(\mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*) \leftarrow \mathcal{A}_2^{\mathsf{QKeyGen}(\cdot),\mathsf{QRandKey}(\cdot),\mathsf{QCor}(\cdot),\mathsf{QSign}(\cdot,\cdot,\cdot)}(\mathsf{st}, \mathsf{mpk})$

Let $i_{\max}$ be the number of queries made to $\mathsf{QKeyGen}(\cdot)$

$S \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}, (\mathcal{QK}_1, \dots, \mathcal{QK}_{i_{\max}}))$

$R \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}^*, (\mathcal{QK}_1, \dots, \mathcal{QK}_{i_{\max}}))$

Let x_S and x_R denote the attributes in case $S, R \neq \perp$

Output: $\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*) = 1 \wedge$

$\Big[\big[\exists (i,j), \mathsf{sk}, x \; \forall (i',j'), \sigma : ((i,j), \mathsf{pk}, \mathsf{sk}, x) \in \mathcal{QK} \setminus \mathcal{QC} \wedge$

$((i',j'), \mathsf{pk}, \mathsf{pk}^*, m^*, \sigma) \notin \mathcal{QS}\big] \vee$

$\big[(S \neq \perp) \wedge (R \neq \perp) \Rightarrow F(x_S, x_R) = 0\big]\Big]$

Figure 2: Unforgeability Game of ULPCS**.**

Attribute-Hiding. The adversary has access to four of the oracles introduced in the previous section: (1) a challenge oracle, to which it can submit an attribute pair (x_0, x_1) and receives as a reply the public key pk corresponding to x_β, for β chosen by the challenger; (2) a rerandomization oracle, to which it can submit indices i and then receives as a reply the rerandomization of the public key that corresponds to this index; (3) a corruption oracle, to which it can submit an index and then receives as a reply the secret key that corresponds to the public key associated with the index; and (4) a signing oracle, to which the adversary can submit an index pair (i,j) as well as a public key pk and a message m and then receives as a reply a signature generated using the j'th rerandomization of the i'th secret key for the public key pk over the message m. The goal of the adversary in this game is to determine the bit β, and thus to observe a difference between the two settings.

To prevent the adversary from trivially winning the game, we need to specify validity requirements that exclude those distinguishing strategies that are simply based on how the system is supposed to operate (i.e., correctness) [59]). First, the adversary is only allowed to ask for the corruption of an index i, if the challenge query for this index consists of the same attribute sets, i.e. $x_0 = x_1$. Second, the adversary is only allowed to ask signing queries for an index (i, j) and receiver key pk such that it holds

[2] It is instructive to observe that such (private-key) detectability relations are also studied in the context of RCCA variants [8].

[3] This is captured by the second part of the or-condition which is a logical implication. true if the key has not been output by the key generation generation oracle.

that $F(x_0, y_0) = F(x_1, y_1)$ where (x_0, x_1) is the i'th key challenge query and (y_0, y_1) are the possible attributes associated with pk. To determine the attributes (y_0, y_1) of pk, we make, again, use of the detectability of the scheme and execute the detection algorithm using pk as an input. The game is described in Figure 3. We formally define attribute-hiding in Appendix B.4.

$\mathsf{AH}_{\beta}^{\mathsf{ULPCS}}(1^{\lambda}, \mathcal{A})$

$(F, \mathsf{st}) \leftarrow \mathcal{A}_1(1^{\lambda})$

$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{ULPCS.Setup}(1^{\lambda}, F)$

$\alpha \leftarrow \mathcal{A}_2^{\mathsf{QKeyGenLR}_{\beta}(\cdot,\cdot),\mathsf{QRandKey}(\cdot),\mathsf{QCor}(\cdot),\mathsf{QSign}(\cdot,\cdot,\cdot)}(\mathsf{st}, \mathsf{mpk})$

Output: α

Figure 3: The Attribute-Hiding game for ULPCS**.**

Unlinkability. Unlinkability captures the property that a party can re-randomize its key such that it is not possible to tell afterwards whether this party is acting again or whether it is another party that freshly joined the system. Coupled with attribute-hiding, this leads to strong privacy guarantees: observing a signature between two freshly re-randomized public keys does not reveal anything beyond the assertion that the attributes behind the keys satisfy the policy without any link to a party's other actions in the system.

The simple single user unlinkability security game (for the case $|\mathcal{QK}| = 1$) in Figure 4 is parameterized by the challenge bit β. It formalizes that an adversary is not able tell apart a user that evolves its key from fresh keys with the same attribute, while all "versions" of the original key are in use to sign off arbitrary messages towards arbitrary recipients (the adversary can create arbitrary users and know their secrets). We show in Appendix C that this intuitive game is sufficient to imply security for the multi-user setting. We present the definition formally in Appendix B.5.

For all of the presented security notions, we also consider its bounded T_{Rand} version, where T_{Rand} is a parameter (polynomially in the security parameter). In this bounded version of the different notions (i.e. T_{Rand}-unforgeability, T_{Rand}-attribute-hiding, T_{Rand}-unlinkable) the adversary is restricted to only making at most T_{Rand} rerandomization queries for each obtained key. Looking ahead, bouding the number of rerandomizations admits solutions based on simpler cryptographic assumptions leading to more efficient schemes, which is one goal of this work. Our concrete schemes achieve T_{Rand} bounded security and in the instantiations we set $T_{\mathsf{Rand}} = 2^{16} - 1$.

$\mathsf{Link}_{\beta}^{\mathsf{ULPCS}}(1^{\lambda}, \mathcal{A})$

$(F, \mathsf{st}_1) \leftarrow \mathcal{A}_1(\lambda)$

$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^{\lambda}, F)$

$(x, \mathsf{st}_2) \leftarrow \mathcal{A}_2^{\mathsf{KeyGen}(\mathsf{msk},\cdot)}(\mathsf{mpk}, \mathsf{st}_1)$

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x)$; $\mathcal{QK} \leftarrow ((1,0), \mathsf{pk}, \mathsf{sk}, x)$

$\alpha \leftarrow \mathcal{A}_3^{\mathsf{KeyGen}(\mathsf{msk},\cdot),\mathsf{QRandKey}_{\beta}(\cdot),\mathsf{QSign}(\cdot,\cdot,\cdot)}(\mathsf{pk}, \mathsf{st}_2)$

Output: α

Figure 4: Single-Challenge Unlinkability game of ULPCS**.**

4 CONSTRUCTIONS

4.1 ul-PCS for Generic Policies

We first present the construction for generic policies in Figure 5. The main idea is to equip a public key with the encryption of the user's attributes. For this we use a predicate-encryption scheme that supports the predicate class $f_x(y) := F(x, y)$, where F is the policy. Crucially, there must be a link to the issuance of attribute x towards a user by the authority (CA). This link is established via pseudorandom identifiers derived from a PseudoRandom Function (PRF) that are developed over the course of key updates. Critically, such key updates are not only bound to the attributes, but also to the functional key of the PE scheme, and to a signature public key—a master public key that grants the user the right to authorize fresh signature public keys to itself, reminiscent of identity-based signature schemes. Those fresh keys can be published as part of a new version of the public key, and signed with the master signature private key. Note that due to the unlinkability requirement, the master signature keys as well as the generated signatures must remain private. The NIZK proof in the public key assures the well-formedness of the key, resulting in fresh-looking re-randomizations that are, however, bound to what an authority granted to a user. Finally, when signing, the sender first checks that the recipient's public key is valid and, then proves eligibility by proving that knowledge of a (certified) functional key sk_{f_x} of the PE scheme that is able to decrypt the ciphertext in the recipient's public key. The final step is signing the message and the proof using the newest signature public key. We require two NIZK languages for our approach: $\mathcal{L}_1$ refers to the language of valid public keys; $\mathcal{L}_2$ refers to the language of eligible signatures:

Language $\mathcal{L}_1$: A statement $x_{\mathsf{st}} := (T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ct}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{A}, \mathsf{mpk}_{\mathsf{PE}})$ is in the language $\mathcal{L}_1$ if it holds for a witness $w_{\mathsf{st}} := (\mathsf{k}, \mathsf{ctr}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, x, \sigma_{\mathsf{sig}}^1, \sigma_{\mathsf{sig}}^2, \sigma_{\mathsf{ctr}})$ that:

- $\mathsf{ctr} < T_{\mathsf{Rand}}$
- $\mathsf{ct}_{\mathsf{ctr}} = \mathsf{PE.Enc}(\mathsf{mpk}_{\mathsf{PE}}, x)$
- $\mathsf{ID}_{\mathsf{ctr}} = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$
- $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^{A}, (\mathsf{k}, x), \sigma_{\mathsf{sig}}^1) = 1$
- $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^{A}, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}), \sigma_{\mathsf{sig}}^2) = 1$
- $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}, (\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ID}_{\mathsf{ctr}}), \sigma_{\mathsf{ctr}}) = 1$

Language $\mathcal{L}_2$: A statement $x_{\mathsf{st}} := (\mathsf{ID}_S, \mathsf{ct}_R, \mathsf{vk}_{\mathsf{sig}}^{A})$ is in the language $\mathcal{L}_2$ if it holds for a witness $w_{\mathsf{st}} := (\mathsf{k}, \mathsf{ctr}, \mathsf{sk}_{f_x}, \sigma_{\mathsf{sig}}^3)$ that:

- $\mathsf{PE.Dec}(\mathsf{sk}_{f_x}, \mathsf{ct}_R) = 1$
- $\mathsf{ID}_S = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$
- $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^{A}, (\mathsf{k}, \mathsf{sk}_{f_x}), \sigma_{\mathsf{sig}}^3) = 1$

The helper function $\mathsf{ValidPK}(\mathsf{mpk}, \mathsf{pk})$ in Figure 5 checks the public-key's well-formedness by verifying the $\mathsf{NIZK}_{\mathcal{L}_1}$ proof of pk and outputs 1 if it verifies, and 0 otherwise. We refer to Appendix E for the formal security analysis that establishes the following:

Theorem 4.1. *The ul-PCS scheme for generic policies based on pseudo-random functions, predicate encryption, unforgeable signatures and extractable NIZK systems for languages $\mathcal{L}_1$ and $\mathcal{L}_2$ is T_{Rand} unforgeable, attribute-hiding, and unlinkable, where T_{Rand} is polynomial in the security parameter.*

Looking ahead, we can pick $T_{\mathsf{Rand}} = 2^{16} - 1$ in our instantiations.

$\underline{\mathsf{Setup}(1^\lambda, F)}$:
$\mathsf{CRS}_{\mathsf{Rand}} \leftarrow \mathsf{NIZK}_{\mathcal{L}_1}.\mathsf{Setup}(1^\lambda)$
$\mathsf{CRS}_{\mathsf{Sign}} \leftarrow \mathsf{NIZK}_{\mathcal{L}_2}.\mathsf{Setup}(1^\lambda)$
$(\mathsf{sk}^A_{\mathsf{sig}}, \mathsf{vk}^A_{\mathsf{sig}}) \leftarrow \mathsf{DS.Setup}(1^\lambda)$
$(\mathsf{mpk}_{\mathsf{PE}}, \mathsf{msk}_{\mathsf{PE}}) \leftarrow \mathsf{PE.Setup}(1^\lambda)$
$\mathsf{mpk} := (T_{\mathsf{Rand}}, F, \mathsf{CRS}_{\mathsf{Rand}}, \mathsf{CRS}_{\mathsf{Sign}}, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}})$
$\mathsf{msk} := (F, \mathsf{sk}^A_{\mathsf{sig}}, \mathsf{msk}_{\mathsf{PE}})$
Return $(\mathsf{mpk}, \mathsf{msk})$
$\underline{\mathsf{KeyGen}(\mathsf{msk}, x)}$:
Parse msk as defined above
$\mathsf{k} \leftarrow \{0,1\}^\lambda$
$(\mathsf{sk}_{\mathsf{sig}}, \mathsf{vk}_{\mathsf{sig}}) \leftarrow \mathsf{DS.Setup}(1^\lambda)$
$\mathsf{sk}_{f_x} \leftarrow \mathsf{PE.KeyGen}(\mathsf{msk}_{\mathsf{PE}}, f_x)$ //$f_x := F(x, \cdot)$
$\sigma^1_{\mathsf{sig}} \leftarrow \mathsf{DS.Sign}(\mathsf{sk}^A_{\mathsf{sig}}, (\mathsf{k}, x)), \sigma^2_{\mathsf{sig}} \leftarrow \mathsf{DS.Sign}(\mathsf{sk}^A_{\mathsf{sig}}, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}))$
$\sigma^3_{\mathsf{sig}} \leftarrow \mathsf{DS.Sign}(\mathsf{sk}^A_{\mathsf{sig}}, (\mathsf{k}, \mathsf{sk}_{f_x}))$
$\mathsf{usk} := (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, \sigma^1_{\mathsf{sig}}, \sigma^2_{\mathsf{sig}}, \sigma^3_{\mathsf{sig}}, x, \mathsf{sk}_{f_x})$
Return $(\mathsf{pk}_0, \mathsf{sk}_0) \leftarrow \mathsf{RandKey}(\mathsf{mpk}, (\mathsf{usk}, -1, \perp))$
$\underline{\mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})}$:
Parse mpk, usk as defined above and $\mathsf{sk} = (\mathsf{usk}, \mathsf{ctr}, \cdot)$
$\mathsf{ctr} := \mathsf{ctr} + 1$
If $\mathsf{ctr} \geq T_{\mathsf{Rand}}$: return $\perp$
$\mathsf{ID}_{\mathsf{ctr}} := \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$
$(\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{sk}^{\mathsf{ctr}}_{\mathsf{sig}}) \leftarrow \mathsf{DS.Setup}(1^\lambda)$
$\sigma_{\mathsf{ctr}} \leftarrow \mathsf{DS.Sign}(\mathsf{sk}_{\mathsf{sig}}, (\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{ID}_{\mathsf{ctr}}))$
$\mathsf{ct}_{\mathsf{ctr}} \leftarrow \mathsf{PE.Enc}(\mathsf{mpk}_{\mathsf{PE}}, x)$
$\pi_{\mathsf{ctr}} \leftarrow \mathsf{NIZK}_{\mathcal{L}_1}.\mathsf{Prove}(\mathsf{CRS}_{\mathsf{Rand}},$
$(T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{ct}_{\mathsf{ctr}}, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}}), (\mathsf{usk}, \sigma_{\mathsf{ctr}}))$
$\mathsf{pk}_{\mathsf{ctr}} := (\mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{ct}_{\mathsf{ctr}}, \pi_{\mathsf{ctr}})$
Return $(\mathsf{pk}_{\mathsf{ctr}}, \mathsf{sk}_{\mathsf{ctr}} := (\mathsf{usk}, \mathsf{ctr}, \mathsf{sk}^{\mathsf{ctr}}_{\mathsf{sig}}))$
$\underline{\mathsf{Sign}(\mathsf{mpk}, \mathsf{sk}, \mathsf{pk}_R, m)}$:
Parse $\mathsf{mpk}, \mathsf{sk} := (\mathsf{usk}, \mathsf{ctr}, \mathsf{sk}_{\mathsf{ctr}})$ and usk as above
If $\mathsf{ValidPK}(\mathsf{mpk}, \mathsf{pk}_R) = 0$: return $\perp$
$\mathsf{ID}_S := \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$
If $\mathsf{PE.Dec}(\mathsf{sk}_{f_x}, \mathsf{ct}_R) = 0$: return $\perp$
$\pi_s \leftarrow \mathsf{NIZK}_{\mathcal{L}_2}.\mathsf{Prove}(\mathsf{CRS}_{\mathsf{Sign}}, (\mathsf{ID}_S, \mathsf{ct}_R, \mathsf{vk}^A_{\mathsf{sig}}), \mathsf{sk})$
$\sigma \leftarrow \mathsf{DS.Sign}(\mathsf{sk}_{\mathsf{ctr}}, (m, \mathsf{pk}_R, \pi_s))$
Return (π_s, σ)
$\underline{\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}_S, \mathsf{pk}_R, m, \sigma)}$:
Parse mpk as defined above and $\sigma = (\pi, \sigma')$
If $\mathsf{ValidPK}(\mathsf{mpk}, \mathsf{pk}_S) = 0$ or $\mathsf{ValidPK}(\mathsf{mpk}, \mathsf{pk}_R) = 0$, return $\perp$
Return $(\mathsf{NIZK}_{\mathcal{L}_2}.\mathsf{Verify}(\mathsf{CRS}_{\mathsf{Sign}}, (\mathsf{pk}_S, \mathsf{pk}_R), \pi)$
$\wedge\ \mathsf{DS.Verify}(\mathsf{vk}_S, (m, \mathsf{pk}_R, \pi), \sigma'))$

Figure 5: Our generic unlinkable PCS scheme.

4.2 ul-PCS for Specific Policy Types

Separable policies. For separable policies, we observe that we can avoid PE and replace it by ordinary encryption which also allows for more efficient proofs. The crucial observation is that in policies of the form $S(x) \wedge R(y)$ the matching part can be separated as well. The scheme that we formally describe in Appendix D.1 for completeness establishes:

Theorem 4.2. *ul-PCS for separable policies is achievable based on pseudo-random functions, IND-CPA-secure encryption, unforgeable signatures and extractable NIZK systems for languages $\mathcal{L}_1$ and $\mathcal{L}_2$, as defined in Appendix D.1.*

Role-based policies. For role-based policies, we observe that for each "recipient", we can accumulate who is allowed to sign toward this recipient. We can use a very weak form of an accumulator (cf. Appendix A.6) realizable by weakly-secure short signatures [14] and suitably pair it with structure preserving signatures on equivalence classes (cf. Appendix A.5) to re-randomize the accumulator value while leaving the inclusion witnesses intact. The policy can be evaluated via an inclusion check which admits an efficient (pairing-based) NIZK. This optimized scheme achieves outsider-security AH for arbitrary role-based matrices, and full attribute hiding for more specific policies such as the equality policy (or any permutation matrix). The scheme given in Appendix D.2 thus establishes:

Theorem 4.3. *ul-PCS scheme for role-based policies is achievable based on pseudo-random functions, structure-preserving signatures on equivalence classes, standard (unforgeable) signatures, a weakly-sound accumulator, and extractable NIZK systems for languages $\mathcal{L}_1$ and $\mathcal{L}_2$, as defined in Appendix D.2.*

5 INSTANTIATIONS AND PERFORMANCE

The full documentation on how we instantiate the scheme and realize the NIZK relations effectively are given in Appendices F and G due to space constraints. Here, we quickly survey the deployed algorithms before reporting on the measured performance.

We require three different types of signature schemes to instantiate the proposed constructions. For structure-preserving signatures on equivalence-classes (SPS-EQ) we use the construction proposed in [39]. For standard digital signatures, we mainly employ BLS signatures [16]. However, if we need a Groth-Sahai (GS) friendly relation, we use the structure-preserving signatures (SPS) proposed in [39]—for example when users need to prove the knowledge of hidden messages and signatures that successfully verify under the verification key of the CA.

We utilize the Dodis-Yampolskiy PseudoRandom Function (PRF) [31] as a well-known and efficient PRF that operates over a cyclic group $\mathbb{G}$ of prime order p.

The proposed generic ul-PCS scheme in Figure 5 relies on PE. We use the Okamoto-Takashima [57] scheme based on dual pairing vector spaces that realizes the inner-product predicate functionality.

The proposed ul-PCS scheme with separable policies relies on public key encryptions for which we use ElGamal encryption [32].

To realize the NIZK relations, we rely on three well-known proof systems: Sigma protocols [60], Groth-Sahai proofs [47], and range-proofs [18]. The syntax of these proof systems can be found in Appendix F. Building all relationships and bridging between the proof systems is far from trivial and we give the full specification (based on which the prototype is based) of how we realize the relations in Appendix G for completeness. A summary of which statements are processed by which proof system is depicted in Figure 6.

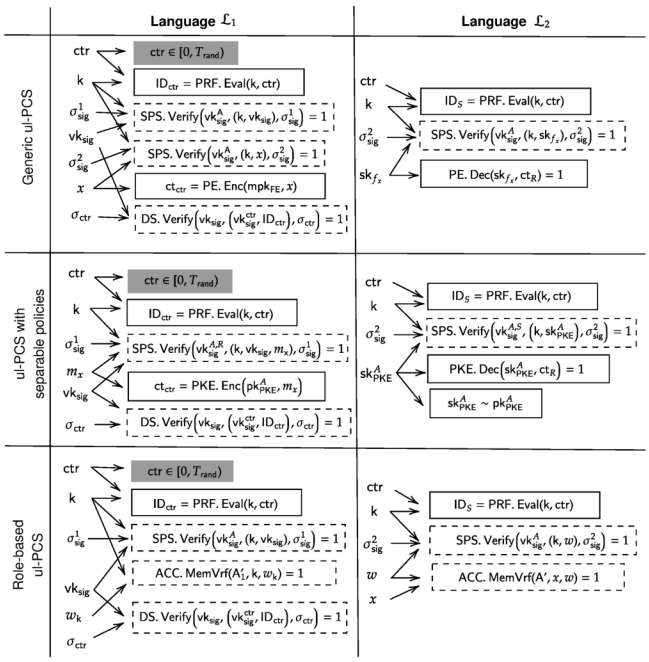


Figure 6: NP-relations and witnesses for our ul-PCS constructions and the used proof systems: Sigma protocols, range-proofs and Groth-Sahai proofs. We use SPS to indicate where we need structure-preserving signatures.

5.1 Performance Analysis

Benchmark & Environment. We implement the proposed ul-PCS schemes and evaluate their performance based on BN-254 elliptic curve groups [10], $y^2 = x^3 + b$, with embedding curve degree 12, where the first group $\mathbb{G}_1$ is a standard curve defined over $\mathbb{Z}_p$. The second group $\mathbb{G}_2$ and target group $\mathbb{G}_T$ are defined over the extension fields $\mathbb{Z}_{p^2}$ and $\mathbb{Z}_{p^{12}}$, respectively. We use the Charm-crypto framework [2] written in Python as the main framework for our open-source implementation.[4] In our experiments, we used a machine that we believe represents typical workloads for ul-PCS. We used an HP Zbook 15 G6 with 16 GB of RAM, an Intel Core i7-9850H CPU @ 2.60GHz, and an SSD for storage, running Ubuntu 22.04 LTS.

Remark 1. *As explained in Section 2.3, our implementation has the primary purpose to establish feasibility together with an initial cost estimation for PCS and to be a basis for future developments. In this regard, we opted for CharmCrypto as a comparably easy framework for prototyping. However, we point out that the impact that the choice of an elliptic curve and the underlying library has on performance is significant. Table 1 lists the average runtime based on 1000 independent measurements for the basic operations in the BN-254 curve using the Charm-Crypto framework. For the purpose of comparison, we have included the running time obtained from the bplib[5] library over BN-256 pairing-friendly curves. Bplib relies on OpenPairing[6] which utilizes OpenSSL as the underlying arithmetic framework for implementing an efficient bilinear pairing over the BN curve. It offers promising indications that a production-ready implementation could achieve significantly faster performance. Just the switch from PBC to OpenPairing alone would result in at least six times faster pairing operations which our schemes heavily rely on. Another path forward would be to develop zk-SNARKs for our specific relations (optimizing mainly for verification times and proof sizes), which is an interesting future avenue.*

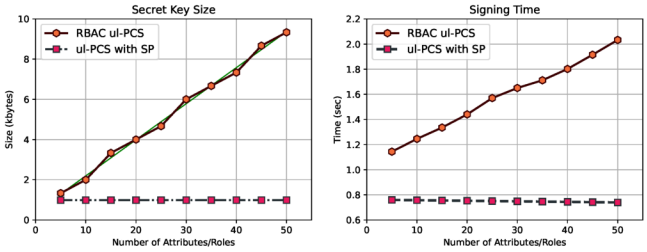
[4]https://github.com/Mahdi171/Unlinkable_PCS.
[5]https://github.com/gdanezis/bplib.
[6]https://github.com/dfaranha/OpenPairing.

Table 1: Cost of basic operations. $\mathbf{M}_i$ and $\mathbf{E}_i$ denote multiplication and exponentiation costs in $\mathbb{G}_i$ and $\mathbf{P}$ denotes pairing cost.

Library/Curve	$\mathbf{M}_1/\mathbf{M}_2/\mathbf{M}_T$ time (μs)	$\mathbf{E}_1/\mathbf{E}_2/\mathbf{E}_T$ time (ms)	$\mathbf{P}$ time (ms)
Charm-Crypto/BN-254	3.3/7.1/21.4	0.9/1.6/4.8	18.5
bplib/BN-256	3.8/6/3	0.3/1/2.3	2.7

Scheme	KeyGen time (ms)	RandKey time (ms)	Verify time (ms)	pk Size (KB)	σ Size (KB)
ul-PCS (role-based)	750	550	1 630	28	16
ul-PCS with separable policies	490	480	1 020	28	14.5

(a) Running time/size of constant operations/parameters



(b) Secret-key size and Signing time

Figure 7: Performance of RBAC and separable ul-PCS.

Optimized constructions for special policies. All computations are performed on the same machine and we achieve practical results, even for large attribute sets and policies. We report the execution times on an average of 100 executions without preprocessing. The maximum number of re-randomizations is assumed to be $T_{\text{Rand}} = 2^{16} - 1$. The length of secret key and signing time are shown in Figure 7b. Figure 7a depicts the constant execution times and parameter sizes. A summary of our analysis is as follows:

In the role-based ul-PCS described in Figure 19, it takes around 1.2 sec. (resp. 2 sec.) to sign a message using a secret key with 5 roles (resp. 50 roles). The average cost per additional attribute is around 19 ms. It takes around 1.6 seconds for a verifier to verify the validity of a signature independent of the number of roles. The required memory for a user to store a secret key representing 5 roles (resp. 50 roles) is not larger than 2 KB (resp. 10 KB) and we pay 270 bytes per additional attribute. The corresponding public key has a constant size of 28 KB, again, independent of the number of roles. A signature in this scheme has a constant size of 16 KB.

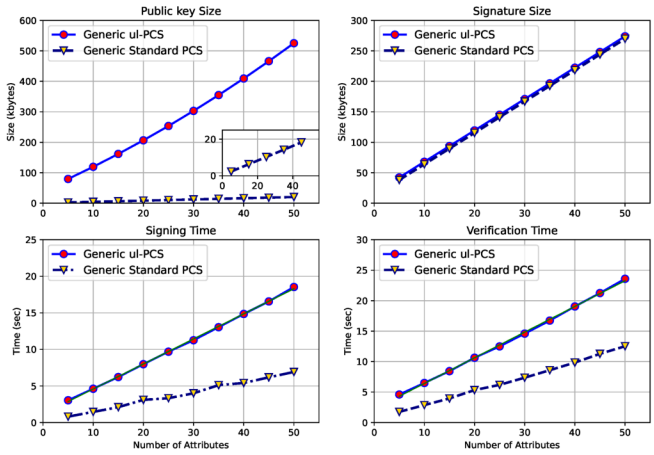


Figure 8: Performance of the generic ul- and standard PCS.

The proposed ul-PCS scheme for separable policies described in Figure 18, achieves a slightly better performance in most operations. More precisely, a secret key can be generated in less than 490 ms independent of the number of attributes. The key re-randomization phase also benefits from this constant running time and requires 480 ms to be executed. Unsurprisingly, as illustrated in Figure 7b, the signing time is also constant and it takes around 750 ms to sign a message. Signature verification takes around 1 second. The secret keys are also constant and require only 1 kbyte of storage while the corresponding public key is 28 KB large. A signature has a constant size of 14.5 KB, which is slightly shorter than for the role-based ul-PCS.

Generic ul-PCS/PCS with IP-PE. In Figure 8, we compare the overhead of the proposed generic ul-PCS scheme with the standard PCS scheme proposed by Badertscher et al. [7]. Here, we use the same inner-product predicate encryption scheme for both prototypes. We are furthermore interested in the dependency on the number of attributes, where attributes are encoded as length-n vectors (over a base field). Due to space constraints, we focus our attention on the "online operations" that are part of both schemes (signing and verification times; signature and public key sizes). We explore the range between 5 and 50 attributes. We observe that standard PCS generally has better performance characteristics, which can be attributed to the cost of unlinkability/anonymity.

A public key for 5 attributes in the generic ul-PCS scheme has a size of around 79 KB and we pay 9.9 KB per additional attribute. In the generic standard PCS scheme we start at 2.3 KB and pay 390 bytes per additional attribute. Signature sizes are almost identical in both schemes, a signature generated in the case of 5 attributes is around 40 KB and each additional attribute incurs a cost of about 5.14 KB. In terms of signing times, in the generic ul-PCS a message can be signed in around 3 seconds (resp. 18.5 seconds) in case of 5 attributes (resp. 50 attributes). The average cost per additional attribute in this range from 5 to 50 attributes can be estimated with 340 ms. For standard PCS, we require about 800 ms (resp. up to 6.9 seconds) to generate a signature for 5 attributes (resp. 50 attributes). The average cost per additional attribute here is 130 ms. Finally, verification of a signature in the context of 5 attributes (resp. 50 attributes) takes about 4.59 seconds (resp. 23 seconds) for the generic ul-PCS scheme. For the standard PCS scheme the verification time is around 1.74 seconds (resp. 12.5 seconds) for 5 (resp. 50) attributes. In this range, the price per additional attribute can be estimated at 420 ms for the generic ul-PCS scheme and 230 ms for the standard PCS scheme.

6 APPLICATIONS

6.1 Enhancing DAP Systems

Overview. We now formally show how unlinkable policy-compliant signatures can be combined with decentralized anonymous payment (DAP) systems such as Monero [3] or Zcash [12], to yield a system that couples credentials and policy checks at the address resp. transaction level. In a recent work, Engelmann et al. [33] (PETS'22) introduced a new abstraction model for protocols like Zcash and Monero to reason about their confidentiality, privacy, and soundness properties. Recall that in such UTxO-like privacy-preserving transaction systems, confidentiality and anonymity of transacted amounts and involved addresses must be ensured, in contrast to plain UTxO-based systems like Bitcoin [55], where a data structure of unspent transaction outputs is maintained by the ledger, where an unspent transaction output can be thought of as an address-value pair and spending is the act of publishing a valid signature (corresponding to the address, typically a hash of a public key).

In privacy-enhanced systems, the output of a transaction is typically encrypted using the public key of the recipient and, by using key-private encryption [12], the transaction outputs become anonymous when posted on a public ledger. To enable the UTxO functionality, the system further must allow the generation of what is often called a "nullifier" value that marks an existing and unique transaction output as spent without disclosing further information. To abstract the concrete mechanisms used in different systems, Engelmann et al. [33] define the notion of a one-time account (OTA) scheme. We recall the definition of an OTA scheme in Appendix I, which is a tuple of algorithms $\mathsf{OTA} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{NoteGen}, \mathsf{Enc}, \mathsf{Receive}, \mathsf{NulEval})$. An OTA scheme is the privacy-preserving analogue and generalization of a plain UTxO based transaction system described above: an OTA scheme allows to generate (knowing the intended recipients' public keys), for each transaction output, a unique and anonymous one-time account which is called a "note". The contents of a note can only be accessed using the recipient's public key, which can be further used to claim it by computing a unique nullifier value that anonymously marks it as spent. If the intended recipient requires auxiliary information to create the nullifier, an OTA scheme has an explicit function to encrypt such values towards the recipient (this ciphertext is formally part of the transaction output and accompanies the note).

Based on an OTA scheme, the main task of a higher-level transaction system is to maintain a ledger, recording notes in its state, whether they have been spent or not, and to implement a certain monetary policy (such as conservation of money during a standard transaction, or how to mint coins in special transactions). This is highly application dependent, and the OTA scheme provides

the core infrastructure underneath. The high-level transaction mechanic is as follows: in a transaction, one declares knowledge of (input) notes contained in the ledger state and presents their nullifiers plus a NIZK proof that they are constructed correctly based on the input notes. Importantly, the transaction reveals no link to the input notes other than their containment in the ledger state (the nullifiers ensure that no note can be spent more than once). Finally, a transaction specifies a new set of output notes, and an application-dependent proof that the output notes stand in a particular relation with the input notes (such as that the sum of all inputs equals the sum of all outputs minus a given fee). We refer to [12] and [33] on how these systems can be constructed based on an OTA scheme.

We now present two constructions how to combine PCS with OTA to achieve accounts that are bundled with private attributes about which policy compliance can be proved. The first construction is the generic composition of PCS and OTA and embeds PCS via the use of recursive SNARKS. The second construction constitutes an efficiency improvement in case the PCS is unlinkable and realizes the vision presented in Section 2.2. Due to space constraints, we defer the description of *Construction I* to Appendix I.2.

Construction II. We present an efficient way to compose the two schemes while retaining essentially the same privacy guarantees as the first construction above by leveraging the unlinkability feature of the ul-PCS scheme. First, we describe the scheme and argue about its security. The scheme works as follows: the sender creates a note according to the OTA scheme and PCS-signs a commitment to the note such that it verifies with the sender's and recipient's current PCS public keys respectively. We leave the format of the note unchanged and transmit the additional information as well as the commitment opening as part of the ciphertext of the OTA scheme. The nullifier on the other hand will be the OTA nullifier, both PCS public keys, the PCS signature on the commitment, plus another PCS signature created by the recipient on the OTA nullifier, and a NIZK that proves knowledge of the opening information of the commitment. (Recall that PCS-Signing requires specifying a target public key which is not relevant at this step. For simplicity, we assume that a party can "sign towards itself", in which case standard signatures are a special case of PCS.) In summary, a party can only claim ownership of a note (by constructing the nullifier) if it possesses the underlying OTA private key (to decrypt the output and to generate the OTA nullifier) *and* possesses the PCS private key that corresponds to the PCS public key towards which the note was created, i.e., for which the signature on the note successfully verifies. We observe that this construction avoids a NIZK about PCS signatures and gets away with just a simple commitment proof. The scheme is formally given below. We note that the only change to the interface is that KeyGen can have black-box access to a PCS key-gen oracle and that it is parameterized by an attribute x. Thanks to this modularity, the OTA security requirements remain well-defined and we give an analysis in Appendix I.3.

Setup: Run $(\mathsf{mpk}, \mathsf{msk}) \leftarrow$ ul-PCS.Setup and $\mathsf{pp} \leftarrow$ OTA.Setup and define the public parameter $\mathsf{p} \leftarrow (\mathsf{mpk}, \mathsf{pp})$. For simplicity, p is implicitly provided to all algorithms below and not explicitly mentioned.

$\mathsf{KeyGen}_x^{\mathsf{KeyGen}_{\mathrm{PCS}}(\mathsf{msk},\cdot)}$: Run $(\mathsf{pk}_{\mathrm{ota}}, \mathsf{sk}_{\mathrm{ota}}) \leftarrow$ OTA.KeyGen and obtain $(\mathsf{pk}_{\mathrm{pcs}}, \mathsf{sk}_{\mathrm{pcs}})$ for attribute x via an oralce call. Define $\mathsf{pk} = (\mathsf{pk}_{\mathrm{ota}}, \mathsf{pk}_{\mathrm{pcs}})$ and $\mathsf{sk} = (\mathsf{sk}_{\mathrm{ota}}, \mathsf{sk}_{\mathrm{pcs}})$.

$\mathsf{NoteGen}((\mathsf{pk}^R_{\mathrm{ota}}, \mathsf{pk}^R_{\mathrm{pcs}}), \vec{a}, (r_1, r_2))$: Run $\mathsf{note} \leftarrow$ OTA.NoteGen$(\mathsf{pk}_{\mathrm{ota}}, \vec{a}, r_1)$.

$\mathsf{Enc}((\mathsf{pk}^R_{\mathrm{ota}}, \mathsf{pk}^R_{\mathrm{pcs}}), \vec{a}, (r_1, r_2), (\mathsf{sk}_{\mathrm{ota}}, \mathsf{sk}_{\mathrm{pcs}}), \xi)$: Re-create the note note using r_1 as above and compute $\mathsf{Com} \leftarrow \mathsf{Commit}(\mathsf{note}; r_2)$. Run $(\mathsf{sk}'_{\mathrm{pcs}}, \mathsf{pk}'_{\mathrm{pcs}}) \leftarrow$ ul-PCS.RandKey$(\mathsf{sk}_{\mathrm{pcs}})$ and store the new PCS keys. Run $\sigma_{\mathrm{note}} \leftarrow$ ul-PCS.Sign$(\mathsf{sk}'_{\mathrm{pcs}}, \mathsf{pk}^R_{\mathrm{pcs}}, \mathsf{Com})$. Compute $C \leftarrow \mathsf{Enc}\big(\mathsf{pk}^R_{\mathrm{ota}}, (\vec{a}, (r_1, r_2), \mathsf{pk}'_{\mathrm{pcs}}, \mathsf{pk}^R_{\mathrm{pcs}}, \sigma_{\mathrm{note}}), \xi\big)$.

$\mathsf{Receive}(\mathsf{note}, C, (\mathsf{sk}_{\mathrm{ota}}, \mathsf{sk}_{\mathrm{pcs}}))$: Compute OTA.Receive$(\mathsf{note}, C, \mathsf{sk}_{\mathrm{ota}})$.

$\mathsf{NulEval}((\mathsf{sk}^R_{\mathrm{ota}}, \mathsf{sk}^R_{\mathrm{pcs}}), \vec{a}, (r_1, r_2), \mathsf{pk}^S_{\mathrm{pcs}}, \mathsf{pk}^R_{\mathrm{pcs}}, \sigma_{\mathrm{note}})$: Verify that $\mathsf{pk}^R_{\mathrm{pcs}}$ is the public key corresponding to $\mathsf{sk}^R_{\mathrm{pcs}}$ (otherwise, abort). Generate $\mathsf{nul}' \leftarrow$ OTA.NulEval(sk, r_1), compute $\sigma_{\mathrm{nul}} \leftarrow$ ul-PCS.Sign$(\mathsf{sk}^R, \mathsf{pk}^R, \mathsf{nul}')$, and recreate the commitment Com (using $\vec{a}$, r_1, and r_2). Check that ul-PCS.Verify$(\mathsf{pk}^S_{\mathrm{pcs}}, \mathsf{pk}^R_{\mathrm{pcs}}, \mathsf{Com}, \sigma_{\mathrm{note}}) = 1$ (otherwise abort). Finally, output $\mathsf{nul} \leftarrow (\mathsf{nul}', \mathsf{Com}, \mathsf{pk}^S_{\mathrm{pcs}}, \mathsf{pk}^R_{\mathrm{pcs}}, \sigma_{\mathrm{note}}, \sigma_{\mathrm{nul}})$.

For this scheme, we require a NIZK for the following language, which is known to admit efficient proof systems [33][Section 5]:

$$L' = \{(st, \mathsf{Com}, \mathsf{nul}') \mid \exists(\mathsf{note}, \mathsf{sk}_{\mathrm{ota}}, \vec{a}, r_1, r_2) : \mathsf{note} \in st \\ \wedge \mathsf{Com} = \mathsf{Commit}(\mathsf{note}, r_2) \wedge \mathsf{note} = \mathsf{NoteGen}(P(\mathsf{sk}_{\mathrm{ota}}), \vec{a}, r_1) \\ \wedge \mathsf{nul}' = \mathsf{NulEval}(\mathsf{sk}_{\mathrm{ota}}, r_1)\}.$$

6.2 Distributed Setup and User Enrollment

In credential systems, issuance is often distributed across a set of servers to avoid a single point of failure. Such failures may for example include the leakage or malicious revelation of the master secret key. Hence, the security of the system is improved if the system's setup values and user enrollment are implemented by distributed processes with the property that only a large collusion of servers would be able to recreate crucial secret values. This is important in our context, because the revelation of the master secret key would (necessarily) limit the achievable level of attribute hiding in practice, as it allows an attacker to self-issue credentials and determine w.r.t. which participants it can generate valid signatures.

In Appendix H, we showcase how our constructions can be implemented in the distributed setting. In general, the idea is to have the master secret-key shared among the servers (plus additional shared randomness), and have a client obtain partial results $r_i \leftarrow \mathsf{KeyGen}(\mathsf{msk}_i, x)$, and perform client-side aggregation to reconstruct the full output of KeyGen. Such a process ensures that, unless a certain threshold of servers collude (e.g. up to $n-1$ in an honest-but-curious scenario), the CA's have no advantage over any other party in the system.

6.3 Compliant Mixing Services

To obfuscate the transaction graph in (plain) UTxO systems, several techniques have been proposed for Bitcoin [40, 54, 62], all of them requiring the ability that a party can generate fresh public keys (i.e., addresses) at will. For example, in a mixer solution like Obscuro [62], parties would communicate privately a new return address to the

mixer, send coins, and the mixer sends back the coins to the shuffled return addresses of many users.

As a special case of Construction II above, those services can be made compliant using unlinkable PCS at the transaction level (coupling spending keys and PCS keys into one address) and thanks to the re-randomization property, this is compatible with such obfuscation techniques. Both users and service providers could thereby be equipped with credentials certifying that they are accredited in their jurisdiction (or have been KYCed) while still profiting from such services.

6.4 Application within Centralized Designs

Due to its low-level nature of being a signature scheme tied to digital credentials, there is a lot of flexibility in the usage of a PCS scheme. For example, if an application requires traceability or revocation, the user is still free to follow standard procedures to register its public key with a PKI to bind it to a real-world identity, or to secret share its private key with revocation servers that would enable traceability. Having a PKI in place can assist in disincentivizing users from sharing private keys, if that is deemed a concern, as well as any standard technique, such as PKI-assured non-transferability, can be used for this purpose [20] just like with ordinary credential or signature systems.

This is relevant for highly regulated applications including central bank digital currencies that seek *comprehensive regulatory compliance* [52], which is often achieved by enabling full anonymity revocation on any transaction of a user when under investigation. However, more recent trends in CBDCs including Platypus [65] present an alternative path for better balancing accountability and privacy by providing unconditional privacy to transactions for which policy compliance can be proven cryptographically. Here, PCS greatly broadens the scope of such assurances by enabling joint predicates over both sender and receiver attributes including specific combinations of age, citizenship, place of residency, more technical attributes like governing (tax) jurisdiction or financial score, and more generally certified attributes by external auditors (cf. the chosen policy classes for separable policies or RBAC, and of course the richer set computable by inner-product predicates).

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A EXTENDED PRELIMINARIES

A.1 Notation

We denote the security parameter by λ and use 1^λ as its unary representation. We call a randomized algorithm $\mathcal{A}$ *probabilistic polynomial time* (PPT) if there exists a polynomial $p(\cdot)$ such that for every input x the running time of $\mathcal{A}(x)$ is bounded by $p(|x|)$. A function $\mathsf{negl}(\lambda)$ is called *negligible* if for every positive polynomial $p(\lambda)$, there exists λ_0 such that for all $\lambda > \lambda_0 \colon \mathsf{negl}(\lambda) < 1/p(\lambda)$. If clear from the context, we sometimes omit λ for improved readability. The set $\{1, \ldots, n\}$ is denoted as $[n]$ for a positive integer n. For the equality check of two elements, we use "=". The assign operator is denoted with ":=", whereas randomized assignment is denoted with $a \leftarrow A$, with a randomized algorithm A and where the randomness is not explicit. If the randomness is explicit, we write $a := A(x; r)$ where x is the input and r is the randomness. For algorithms $\mathcal{A}$ and $\mathcal{B}$, we write $\mathcal{A}^{\mathcal{B}(\cdot)}(x)$ to denote that $\mathcal{A}$ gets x as an input and has black-box oracle access to $\mathcal{B}$, that is, the response for an oracle query q is $\mathcal{B}(q)$.

A.2 Bilinear Group Setup

Some of our schemes require a bilinear group setup. We use multiplicative notation to refer to group operations.

Definition A.1 (Bilinear Groups [15]). An asymmetric bilinear group generator $\mathcal{BG}(1^\lambda)$ returns a tuple $\mathsf{pp} := (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, \mathrm{G}_1, \mathrm{G}_2)$, such that $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ are cyclic groups of the same prime order p, $\mathrm{G}_1 \in \mathbb{G}_1$ and $\mathrm{G}_2 \in \mathbb{G}_2$ are the generators, and $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is an efficiently computable bilinear pairing with the following properties;

- *non-degeneracy*: $e(\mathrm{G}_1, \mathrm{G}_2) \neq 1_{\mathbb{G}_T}$,
- *bilinearity*: $\forall\, a, b \in \mathbb{Z}_p : e(\mathrm{G}_1^a, \mathrm{G}_2^b) = e(\mathrm{G}_1, \mathrm{G}_2)^{ab} = e(\mathrm{G}_1^b, \mathrm{G}_2^a)$.

Throughout this work, we rely on Type-III bilinear groups for the distinct cyclic groups $\mathbb{G}_1 \neq \mathbb{G}_2$, where there is no efficient algorithm to compute a nontrivial homomorphism in both directions [41]. This type is known as the most efficient choice.

A.3 Pseudorandom Functions

We recall the definition of a pseudorandom function (PRF) as it has been defined in [43].

Definition A.2 (Pseudorandom Function). A pseudo-random function is a keyed function $\mathsf{PRF} : \{0,1\}^\lambda \times \mathcal{X} \to \mathcal{Y}$, where evaluation is done via an efficient algorithm $\mathsf{PRF.Eval}(\mathsf{k}, x)$. For $\beta \in \{0,1\}$, we define the experiment $\mathrm{IND}_\beta^{\mathsf{PRF}}$ in Figure 9, where the oracle O is defined as:

$$O(x) = \begin{cases} \mathsf{PRF.Eval}(\mathsf{k}, x) & \text{if } \beta = 0 \\ \mathsf{RF}(x) & \text{if } \beta = 1 \end{cases} .$$

with $\mathsf{RF}(x)$ denoting a random function. We define the advantage of an adversary $\mathcal{A}$ in the following way:

$$\mathsf{Adv}^{\mathsf{IND}}_{\mathsf{PRF},\mathcal{A}}(\lambda) = |\Pr[\mathrm{IND}_0^{\mathsf{PRF}}(\lambda, \mathcal{A})] - \Pr[\mathrm{IND}_1^{\mathsf{PRF}}(\lambda, \mathcal{A})]| \ .$$

A pseudorandom function PRF is secure, if for any polynomial-time adversary $\mathcal{A}$, there exists a negligible function negl such that: $\mathsf{Adv}^{\mathsf{IND}}_{\mathsf{PRF},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$.

$\mathrm{IND}_\beta^{\mathsf{PRF}}(\lambda, \mathcal{A})$
$\mathsf{k} \leftarrow \{0,1\}^\lambda$
$\alpha \leftarrow \mathcal{A}^{O(\cdot)}(1^\lambda)$
Output: α

Figure 9: Security Game for PRF

A.4 Digital Signatures

We recap the definition of digital signatures as well as existential unforgeability [45].

Definition A.3 (Digital Signatures). A digital signature scheme (DS) is a triple of PPT algorithms $\mathsf{DS} = (\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$, defined as follows:

- $\mathsf{Setup}(1^\lambda)$: Takes as input a unary representation of the security parameter λ and outputs a verification key vk and a signing key sk.
- $\mathsf{Sign}(\mathsf{sk}, m)$: Takes as input the signing key sk, a message $m \in \mathcal{M}$ and outputs a signature σ.
- $\mathsf{Verify}(\mathsf{vk}, m, \sigma)$: Takes as input the verification key vk, a message m and a signature σ, and outputs 0 or 1.

A scheme DS is *correct* if (for all $\lambda \in \mathbb{N}$), for all vk in the support of $\mathsf{Setup}(1^\lambda)$ and all $m \in \mathcal{M}$, we have

$$\Pr[\mathsf{Verify}(\mathsf{vk}, m, \mathsf{Sign}(\mathsf{sk}, m)) = 1] = 1.$$

Definition A.4 (Existential Unforgeability). Let $\mathsf{DS} = (\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$ be a DS scheme. We define the experiment $\mathsf{EUF\text{-}CMA}^{\mathsf{sig}}$ in Figure 10 with Q being the set containing the queries of $\mathcal{A}$ to the signing oracle $\mathsf{Sign}(\mathsf{sk}, \cdot)$. The advantage of an adversary $\mathcal{A}$ is defined by

$$\mathsf{Adv}^{\mathsf{EUF\text{-}CMA}}_{\mathsf{DS},\mathcal{A}}(\lambda) = \Pr[\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}(1^\lambda, \mathcal{A}) = 1].$$

A Digital Signature scheme DS is called *existentially unforgeable under adaptive chosen-message attacks (EUF-CMA secure)* if for any polynomial-time adversary $\mathcal{A}$ it holds that $\mathsf{Adv}^{\mathsf{EUF\text{-}CMA}}_{\mathsf{DS},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$ for a negligible function $\mathsf{negl}(\cdot)$.

$\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}(1^\lambda, \mathcal{A})$
$(\mathsf{vk}, \mathsf{sk}) \leftarrow \mathsf{Setup}(1^\lambda)$
$(m, \sigma) \leftarrow \mathcal{A}^{\mathsf{Sign}(\mathsf{sk}, \cdot)}(\mathsf{vk})$
Output: $\mathsf{Verify}(\mathsf{vk}, m, \sigma) = 1 \wedge m \notin Q$

Figure 10: Existentially Unforgeability for signatures.

A.5 Structure-Preserving Signatures on Equivalence Classes.

Structure-Preserving Signatures (SPS) [1] are a special type of digital signatures defined over bilinear groups that fulfill certain extra properties. More precisely, the verification key, message and signature are only source group elements and, to verify the validity of a signature, only group membership checks and pairing product

equations are allowed. SPS have the same algorithm as digital signatures as defined in Definition A.3 and guarantee unforgeability as defined in Definition A.4.

SPS on Equivalence classes (SPS-EQ) proposed by Hanser and Slamanig [48] are special type of SPS that enable joint re-randomization of signatures and the signed messages. SPS-EQ provide a controlled form of malleability such that one can change the representation of the message and the corresponding signature. More precisely, for a given prime-order group $\mathbb{G}$ we can define a projective vector $(\mathbb{G}^*)^\ell$ based on the following relation, where $\ell > 1$ and $\mathbb{G}^*$ denotes the set of all group elements without the identity element of the group.

$$\mathcal{R} := \left\{ (\vec{M}, \vec{M}^*) \in (\mathbb{G}^*)^\ell \times (\mathbb{G}^*)^\ell \mid \exists\, \mu \in \mathbb{Z}_p^* \text{ s.t. } \vec{M}^* = \vec{M}^\mu \right\} . \tag{1}$$

This is an equivalence relation for prime order groups.
The equivalence class of a vector $\vec{M} \in (\mathbb{G}^*)^\ell$ for some $\ell > 1$ is defined by:

$$[\vec{M}]_{\mathcal{R}} := \{ \vec{M}^* \in (\mathbb{G}^*)^\ell \mid (\vec{M}, \vec{M}^*) \in \mathcal{R} \} .$$

The Class-hiding property of equivalence classes guarantees that it is computationally hard to distinguish elements of the same equivalence class from randomly sampled group elements.

Definition A.5 (Class-Hiding [48]). A relation $\mathcal{R}$ is called class-hiding if for all PPT adversaries, $\mathcal{A}$, and $\ell > 1$ we have:

$$\left| \Pr \left[\begin{array}{l} \vec{M} \xleftarrow{\$} (\mathbb{G}^*)^\ell, \vec{M}_0 \xleftarrow{\$} (\mathbb{G}^*)^\ell, \vec{M}_1 \xleftarrow{\$} [\vec{M}]_{\mathcal{R}}, \\ b \xleftarrow{\$} \{0,1\}, b' \leftarrow \mathcal{A}(\vec{M}, \vec{M}_b) \mid b = b' \end{array} \right] - \frac{1}{2} \right| \leq \mathsf{negl}(\lambda)$$

Hanser and Slamanig [48] formally prove that, as long as DDH is hard, the relation described in equation 1 is class-hiding. We only consider this relation in this work. In our bilinear setting, the message is based on the second group $\mathbb{G}_2$, but we present the scheme in its general form:

Definition A.6 (Structure-Preserving Signatures on Equivalence classes [48]). In an asymmetric bilinear group, a structure preserving signature over (message space) $(\mathbb{G}_i^*)^\ell$ consists of the following PPT algorithms:

- $\mathsf{Setup}_{\mathcal{R}}(1^\lambda)$: The setup algorithm is a probabilistic algorithm that takes the security parameter λ in its unary representation as input. It outputs public parameters pp as well as an asymmetric bilinear group.
- $\mathsf{KeyGen}_{\mathcal{R}}(\mathsf{pp}, \ell)$: The key generation algorithm is a probabilistic algorithm that takes the public parameters pp and a vector length $\ell > 1$ as inputs. It outputs the key-pair $(\mathsf{sk}, \mathsf{vk})$.
- $\mathsf{Sign}_{\mathcal{R}}(\mathsf{pp}, \mathsf{sk}, \vec{M})$: The signing algorithm is a probabilistic algorithm that takes public parameters pp, secret key sk and a representative message $\vec{M} \in (\mathbb{G}_i^*)^\ell$ for class $[\vec{M}]_{\mathcal{R}}$ as inputs. It outputs the signature σ on message $\vec{M}$.
- $\mathsf{Verify}_{\mathcal{R}}(\mathsf{pp}, \mathsf{vk}, \vec{M}, \sigma)$: The verification algorithm is a deterministic algorithm that takes public parameters pp, a representative message $\vec{M} \in (\mathbb{G}_i^*)^\ell$, a signature σ and a verification key vk as inputs. It then outputs 1 if σ is a valid signature on $\vec{M}$ and 0 otherwise.
- $\mathsf{ChgRep}_{\mathcal{R}}(\mathsf{pp}, \vec{M}, \sigma, \mu, \mathsf{vk})$: The change representation algorithm is a probabilistic algorithm and takes public parameters pp, a representative message $\vec{M} \in (\mathbb{G}_i^*)^\ell$, a signature σ, a scalar $\mu \in \mathbb{Z}_p^*$ and the verification key vk as inputs. It outputs a randomized signature σ' on a new representative message $\vec{M}' = \vec{M}^\mu$.

Since in our work all keys are honestly generated we omit the specification of the function that checks whether a private key is consistent with a given public key (since this holds for honestly generated key pairs).

The primary security requirements for a SPS-EQ scheme are *correctness* and *existential unforgeability against chosen message attack*, which are defined as follows:

Definition A.7 (Correctness). A SPS-EQ scheme over $(\mathbb{G}_i^*)^\ell$ is called *correct*, if the following holds with overwhelming probability for a valid setup pp, any message $\vec{M} \in (\mathbb{G}_i^*)^\ell$, any (valid) key pair $(\mathsf{sk}, \mathsf{pk})$ in the support of $\mathsf{KeyGen}_{\mathcal{R}}(\mathsf{pp}, \ell)$, and any scalar $\mu \in \mathbb{Z}_p^*$:

$$\Pr \left[\begin{array}{l} \mathsf{Verify}_{\mathcal{R}} \left(\mathsf{pp}, \mathsf{vk}, \vec{M}, \mathsf{Sign}_{\mathcal{R}}(\mathsf{pp}, \mathsf{sk}, \vec{M}) \right) = 1 \wedge \\ \mathsf{Verify}_{\mathcal{R}}(\mathsf{pp}, \mathsf{vk}, \vec{M}^\mu, \\ \qquad \mathsf{ChgRep}_{\mathcal{R}}(\vec{M}, \mathsf{Sign}_{\mathcal{R}}(\mathsf{pp}, \mathsf{sk}, \vec{M}), \mu, \mathsf{vk})) = 1 \end{array} \right] .$$

Definition A.8 (Existential Unforgeability). A SPS-EQ over $(\mathbb{G}_i^*)^\ell$ is called adaptively EUF-CMA-secure if for all PPT adversaries $\mathcal{A}$ with access to the signing oracle O_{Sign} we have:

$$\Pr \left[\begin{array}{l} \mathsf{pp} \leftarrow \mathsf{Setup}_{\mathcal{R}}(1^\lambda), (\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{KeyGen}_{\mathcal{R}}(\mathsf{pp}, \ell), \\ \left(\vec{M}^*, \sigma^* \right) \leftarrow \mathcal{A}^{O_{\mathsf{Sign}}}(\mathsf{pp}, \mathsf{vk}) : \\ \forall\, \vec{M} \in Q^{\mathsf{Sign}} : [\vec{M}^*]_{\mathcal{R}} \neq [\vec{M}]_{\mathcal{R}} \\ \qquad \wedge\ \mathsf{Verify}_{\mathcal{R}} \left(\mathsf{pp}, \mathsf{vk}, \vec{M}, \sigma^* \right) = 1 \end{array} \right] \leq \mathsf{negl}(\lambda) ,$$

where the signing oracle O_{Sign} takes a message $\vec{M} \in (\mathbb{G}_i^*)^\ell$ as input, outpus $\mathsf{Sign}_{\mathcal{R}}(\mathsf{pp}, \mathsf{sk}, \vec{M})$ and adds the message to the query set Q^{Sign}.

Finally, we require signature adaptation which shows that signature strings can be perfectly randomized (and thus made unlinkable).

Definition A.9 (Signature Adaptation). An SPS-EQ scheme over $(\mathbb{G}_i^*)^\ell$ perfectly adapts signatures if for all tuples $(\mathsf{sk}, \mathsf{pk}, \vec{M}, \sigma, \mu)$, where $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{pp}, \ell)$, $\vec{M} \in (\mathbb{G}_i^*)^\ell$ and $\mathsf{Verify}(\mathsf{pp}, \mathsf{vk}, \vec{M}, \sigma) = 1$, the two distributions $\mathsf{Sign}(\mathsf{pp}, \mathsf{sk}, \vec{M}^\mu)$ and $\mathsf{ChgRep}_{\mathcal{R}}(\mathsf{pp}, \vec{M}, \sigma, \mathsf{vk}, \mu)$ are identical.

A.6 A Weak Positive Accumulator

We recall an accumulator construction proposed by Karantaidou and Baldimtsi [50] and consider it in the asymmetric bilinear group setting. The construction is derived from Boneh-Boyen signatures [14] and is based on the q-SDH assumption. We only need to consider the positive accumulator, and thus the accumulator value remains constant. In fact, in our application, we only need to guarantee soundness of the accumulator against a weak adversary. As we show below, the soundness requirement of the accumulator

corresponds to what is defined as weakly-unforgeable in [14] for the signature scheme. The witnesses are of constant size, independent of the number of elements in the accumulator set and, additionally, the membership witnesses, after adding new elements, do not need to be updated. In fact, the public accumulator will be set to be the "public key" of the signature scheme and hence does not leak any information about the added elements. The simple accumulator we need can be defined by the following PPT algorithms for the bilinear group setting, where $\mathbb{G}_1 = \langle \mathrm{G}_1 \rangle$, $\mathbb{G}_2 = \langle \mathrm{G}_2 \rangle$. The public parameters are $\mathsf{pp} = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, \mathrm{G}_1, \mathrm{G}_2, e)$.

- ACC.Create(pp): Sample $\alpha \xleftarrow{\$} \mathbb{Z}_p^*$ and define $\mathsf{A} \leftarrow \mathrm{G}_2^{\alpha}$ and $\mathsf{msk} \leftarrow \alpha$ and return $(\mathsf{A}, \mathsf{msk})$. The accumulator domain is $\mathcal{D} = \mathbb{Z}_p \setminus \{\alpha\}$.
- ACC.Add(pp, A, msk, x): To add a new element $x \in \mathcal{D}$ to the accumulator, parse $\mathsf{msk} = \alpha \in \mathbb{Z}_p^*$ and check that $\mathsf{A} = \mathrm{G}_2^{\alpha}$. If the check succeeds compute and return the witness $w_x = \mathrm{G}_1^{1/(x+\alpha)}$.
- ACC.MemVrf(pp, A, x, w_x): If the equation $e(\mathrm{G}_1, \mathrm{G}_2) = e(w_x, \mathrm{AG}_2^x)$ holds, return 1 to approve the membership of x in the accumulator with value A; otherwise output 0 to reject it.

It is straightforward to see that the accumulator is correct. For our purposes, the accumulator has to satisfy the weak soundness notion w.r.t. public parameters pp as we define it in Figure 11. Note that this is a definition tailored to our problem which simplifies the proof of the overall scheme.

$\mathsf{W\text{-}SND}(\mathsf{pp}, \mathsf{ACC}, \mathcal{A})$

$(x_1, \ldots, x_q, \mathsf{st}) \leftarrow \mathcal{A}(\mathsf{pp})$

$(\mathsf{A}, \mathsf{msk}) \leftarrow \mathsf{Setup}(\mathsf{pp})$

Compute for all $i \in [q]$: $\pi_i \leftarrow \mathsf{ACC.Add}(\mathsf{pp}, \mathsf{A}, \mathsf{msk}, x_i)$

$(x^*, \pi^*) \leftarrow \mathcal{A}(\mathsf{st}, \mathsf{A}, (\pi_1, \ldots, \pi_q))$

return $\left(\forall i : x^* \neq x_i\right) \wedge \left(e(\mathrm{G}_1, \mathrm{G}_2) = e(\pi^*, \mathrm{AG}_2^{x^*})\right)$

Figure 11: A weak soundness notion for the accumulator.

We state the following lemma relating the concrete security of q-SDH to the (concrete) winning probability of the above game.

Lemma A.10. *Let* $\mathsf{pp} = (p, \mathrm{G}_1, \mathrm{G}_2, e)$ *be the public parameters. For any PPT adversary* $\mathcal{A}$*, asking at most q queries, that wins the game* $\mathsf{W\text{-}SND}(\mathsf{pp}, \mathsf{ACC}, \mathcal{A})$ *with probability* ε*, there is a PPT adversary* $\mathcal{A}'$ *that on input the q-SDH instance* $(\mathrm{G}_1, y\mathrm{G}_1, \ldots, y^{q'}\mathrm{G}_1, \mathrm{G}_2, y\mathrm{G}_2)$*, where* $y \in \mathbb{Z}_p^*$ *is sampled uniformly at random, returns a valid solution* $(c, (y+c)^{-1}\mathrm{G}_1)$ *for some* $c \in \mathbb{Z}_p \setminus \{-y\}$*, with probability* ε *as long as* $q \leq q'$ *(where the probability is taken over the random choice of y and the internal randomness of* $\mathcal{A}'$*).*

Proof. The proof follows directly from the security proof of the weakly-secure short signature scheme in [14] (version 2014) by observing that ACC is just the weakly-secure signature scheme in disguise, and that our soundness notion perfectly matches the notion of weak unforgeability of [14]. □

Note that, since the statement holds for any concrete set of parameters, it also holds over any distribution of parameters and thus we obtain the asymptotic statement that the accumulator is sound, except with negligible probability in λ under the q-SDH assumption, relative to the bilinear group generation algorithm $\mathcal{BG}(1^\lambda)$ generating the parameters pp.

A.7 Public-Key Encryption

Now, we introduce public-key encryption, together with the notion of IND-CPA security.

Definition A.11 (Public-Key Encryption). A *public-key encryption* (PKE) scheme is a tuple of three algorithms $\mathsf{PKE} = (\mathsf{Setup}, \mathsf{Enc}, \mathsf{Dec})$:

- $\mathsf{Setup}(1^\lambda)$: Takes as input a unary representation of the security parameter λ and outputs a public key pk and a secret key sk.
- $\mathsf{Enc}(\mathsf{pk}, m)$: Takes as input the public key pk and a message $m \in \mathcal{M}$, and outputs a ciphertext ct.
- $\mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$: Takes as input the secret key sk and a ciphertext ct and outputs a message m' or $\perp$.

A public-key encryption scheme PKE is *correct* if for all $\lambda \in \mathbb{N}$, and for all key-pairs $(\mathsf{pk}, \mathsf{sk})$ in the support of $\mathsf{Setup}(1^\lambda)$, we have

$$\Pr[\mathsf{Dec}(\mathsf{sk}, \mathsf{Enc}(\mathsf{pk}, m)) = m] = 1.$$

In this work, we give the adversary access to an encryption challenge oracle that can be queried using multiple challenge message pairs (m_0, m_1). This security definition follows from the standard security definition for a single challenge query using a simple hybrid argument.

Definition A.12 (Indistinguishability-Based Chosen-Plaintext Security). Let $\mathsf{PKE} = (\mathsf{Setup}, \mathsf{Enc}, \mathsf{Dec})$ be a PKE scheme as defined above. For $\beta \in \{0, 1\}$, we define the experiment $\mathsf{IND\text{-}CPA}_\beta^{\mathsf{PKE}}$ in Figure 12, where the left-or-right oracle is defined as:

$\mathsf{QEncLR}_\beta(\cdot, \cdot)$**:** On input two messages m_0 and m_1, output $\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, m_\beta)$.

The advantage of an adversary $\mathcal{A}$ is defined as:

$$\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{A}}(\lambda) = |\Pr[\mathsf{IND\text{-}CPA}_0^{\mathsf{PKE}}(1^\lambda, \mathcal{A}) = 1] - \Pr[\mathsf{IND\text{-}CPA}_1^{\mathsf{PKE}}(1^\lambda, \mathcal{A}) = 1]|.$$

A predicate-only predicate encryption scheme PKE is called *IND-CPA secure* if for any valid polynomial-time adversary $\mathcal{A}$, there exists a negligible function negl such that $\mathsf{Adv}^{\mathsf{IND\text{-}CPA}}_{\mathsf{PKE},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$.

$\mathsf{IND\text{-}CPA}_\beta^{\mathsf{PKE}}(1^\lambda, \mathcal{A})$

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{Setup}(1^\lambda)$

$\alpha \leftarrow \mathcal{A}^{\mathsf{QEncLR}_\beta(\cdot,\cdot)}(\mathsf{pk})$

Output: α

Figure 12: IND-CPA security game of PKE**.**

A.8 Predicate Encryption

To allow for oblivious policy evaluations, we also recap the notion of *predicate-only predicate encryption* as it has been introduced in Katz et al. [51].

Definition A.13 (Predicate-Only Predicate Encryption). Let $\mathcal{F} = \{\mathcal{F}_\lambda\}_{\lambda\in\mathbb{N}}$ be a family of sets $\mathcal{F}_\lambda$ of predicates $f\colon \mathcal{X}_\lambda \to \{0,1\}$. A *predicate-only predicate encryption* (PE) scheme for the functionality class $\mathcal{F}_\lambda$ is a tuple of four algorithms PE = (Setup, KeyGen, Enc, Dec):

- $\mathsf{Setup}(1^\lambda)$: Takes as input a unary representation of the security parameter λ and outputs the master public key mpk and the master secret key msk.
- $\mathsf{KeyGen}(\mathsf{msk}, f)$: Takes as input the master secret key msk and a function $f \in \mathcal{F}$, and outputs a functional key sk_f.
- $\mathsf{Enc}(\mathsf{mpk}, x)$: Takes as input the master public key mpk and an attribute $x \in \mathcal{X}_\lambda$, and outputs a ciphertext ct.
- $\mathsf{Dec}(\mathsf{sk}_f, \mathsf{ct})$: Takes as input a functional key sk_f and a ciphertext ct and outputs 0 or 1.

A predicate-only predicate encryption scheme PE is *correct* if for all $\lambda \in \mathbb{N}$, for all (mpk, msk) in the support of $\mathsf{Setup}(1^\lambda)$, all functions $f \in \mathcal{F}_\lambda$, all secret keys sk_f in the support of $\mathsf{KeyGen}(\mathsf{msk}, f)$, and for all attributes $x \in \mathcal{X}_\lambda$, we have

$$\Pr\big[\mathsf{Dec}(\mathsf{sk}_f, \mathsf{Enc}(\mathsf{mpk}, x)) = f(x)\big] = 1.$$

In the initial work of Katz et al. [51], the authors only introduce the notion of selective security. The corresponding indistinguishability based adaptive security notion for predicate encryption has been introduced in [57]. We present a modification of this definition where the adversary has access to a challenge oracle to which it can submit multiple challenges instead of being able to only submit a single challenge. This security definition directly follows from the standard security definition using a simple hybrid argument.

Definition A.14 (Indistinguishability-Based Attribute Hiding). Let PE = (Setup, KeyGen, Enc, Dec) be a PE scheme for a function family $\mathcal{F} = \{\mathcal{F}_\lambda\}_{\lambda\in\mathbb{N}}$ as defined above. For $\beta \in \{0,1\}$, we define the experiment $\mathsf{AH}^{\mathsf{PE}}_\beta$ in Figure 13, where the left-or-right oracle is defined as:

$\mathsf{QEncLR}_\beta(\cdot,\cdot)$**:** On input two attribute sets x_0 and x_1, output $\mathsf{ct} \leftarrow \mathsf{Enc}(\mathsf{msk}, x_\beta)$.

The advantage of an adversary $\mathcal{A}$ is defined as:

$$\mathsf{Adv}^{\mathsf{AH}}_{\mathsf{PE},\mathcal{A}}(\lambda) = |\Pr[\mathsf{AH}^{\mathsf{PE}}_0(1^\lambda, \mathcal{A}) = 1] - \Pr[\mathsf{AH}^{\mathsf{PE}}_1(1^\lambda, \mathcal{A}) = 1]|.$$

We call an adversary *valid* if for all queries (x_0, x_1) to the oracle $\mathsf{QEncLR}_\beta(\cdot,\cdot)$ and for any function f queried to the key generation oracle $\mathsf{KeyGen}(\mathsf{msk},\cdot)$, we have $f(x_0) = f(x_1)$ (with probability 1 over the randomness of the adversary and the involved algorithms).

A predicate-only predicate encryption scheme PE is called *attribute hiding* if for any valid polynomial-time adversary $\mathcal{A}$, there exists a negligible function negl such that $\mathsf{Adv}^{\mathsf{AH}}_{\mathsf{PE},\mathcal{A}}(\lambda) \le \mathsf{negl}(\lambda)$.

$\mathsf{AH}^{\mathsf{PE}}_\beta(1^\lambda, \mathcal{A})$
$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda)$
$\alpha \leftarrow \mathcal{A}^{\mathsf{KeyGen}(\mathsf{msk},\cdot),\mathsf{QEncLR}_\beta(\cdot,\cdot)}(\mathsf{mpk})$
Output: α

Figure 13: Attribute-Hiding game of PE**.**

A.9 Non-interactive Zero-Knowledge Proofs

In this section, we introduce the notion of non-interactive zero knowledge (NIZK) proofs [11, 38, 44].

Definition A.15 (Non-Interactive Zero-Knowledge Proofs). Let R be an NP Relation and consider the language $L = \{x \mid \exists\, w \text{ with } (x,w) \in R\}$ (where x is called a statement or instance). A non-interactive zero-knowledge proof (NIZK) for the relation R is a triple of PPT algorithms NIZK = (Setup, Prove, Verify):

- $\mathsf{Setup}(1^\lambda)$: Takes as input the unary representation of the security parameter λ and outputs a common reference string CRS.
- $\mathsf{Prove}(\mathsf{CRS}, x, w)$: Takes as input the common reference string CRS, a statement x and a witness w, and outputs a proof π.
- $\mathsf{Verify}(\mathsf{CRS}, x, \pi)$: Takes as input the common reference string CRS, a statement x and a proof π, and outputs 0 or 1.

A system NIZK is complete, if (for all $\lambda \in \mathbb{N}$), for all CRS in the support of $\mathsf{Setup}(1^\lambda)$ and all statement-witness pairs in the relation $(x,w) \in R$,

$$\Pr[\mathsf{Verify}(\mathsf{CRS}, x, \mathsf{Prove}(\mathsf{CRS}, x, w)) = 1] = 1.$$

Besides completeness, a NIZK system should also fulfill the notions of soundness and zero-knowledge, which we introduce in the following two definitions:

$\mathsf{ZK}^{\mathsf{NIZK}}_0(1^\lambda, \mathcal{A}, \mathcal{S})$	$\mathsf{ZK}^{\mathsf{NIZK}}_1(1^\lambda, \mathcal{A}, \mathcal{S})$
$\mathsf{CRS} \leftarrow \mathsf{Setup}(1^\lambda)$	$(\mathsf{CRS}, \tau) \leftarrow \mathcal{S}_1(1^\lambda)$
$\alpha \leftarrow \mathcal{A}^{\mathsf{Prove}(\mathsf{CRS},\cdot,\cdot)}(\mathsf{CRS})$	$\alpha \leftarrow \mathcal{A}^{\mathcal{S}'(\mathsf{CRS},\tau,\cdot,\cdot)}(\mathsf{CRS})$
Output: α	Output: α

Figure 14: Zero-knowledge property of NIZK**.**

Definition A.16 (Zero-Knowledge). Let NIZK = (Setup, Prove, Verify) be a NIZK proof system for a relation R and the corresponding language L, $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$ a pair of algorithms (the simulator), with $\mathcal{S}'(\mathsf{CRS}, \tau, x, w) = \mathcal{S}_2(\mathsf{CRS}, \tau, x)$ for $(x,w) \in R$, and $\mathcal{S}'(\mathsf{CRS}, \tau, x, w) = \text{failure}$ for $(x,w) \notin R$. For $\beta \in \{0,1\}$, we define the experiment $\mathsf{ZK}^{\mathsf{NIZK}}_\beta(1^\lambda, \mathcal{A})$ in Figure 14. The associated advantage of an adversary $\mathcal{A}$ is defined as

$$\mathsf{Adv}^{\mathsf{ZK}}_{\mathsf{NIZK},\mathcal{A},\mathcal{S}}(\lambda) := |\Pr[\mathsf{ZK}^{\mathsf{NIZK}}_0(1^\lambda, \mathcal{A}, \mathcal{S}) = 1] - \Pr[\mathsf{ZK}^{\mathsf{NIZK}}_1(1^\lambda, \mathcal{A}, \mathcal{S}) = 1]| \ .$$

A NIZK proof system NIZK is called perfect zero-knowledge, with respect to a simulator $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$, if $\mathsf{Adv}^{\mathsf{ZK}}_{\mathsf{NIZK},\mathcal{A},\mathcal{S}}(\lambda) = 0$

$\underline{\mathsf{CORR}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A})}$
$(F, \mathsf{st}) \leftarrow \mathcal{A}_1(1^\lambda)$
$(\mathsf{msk}, \mathsf{mpk}) \leftarrow \mathsf{Setup}(1^\lambda, F)$
$c \leftarrow 0$
$\left(\begin{smallmatrix}(i,j),\\(k,\ell)\end{smallmatrix}, m\right) \leftarrow \mathcal{A}_2^{\mathcal{O}_{\mathsf{Gen}}, \mathcal{O}_{\mathsf{ReRand}}}(\mathsf{st}, \mathsf{mpk})$
$(\mathsf{sk}_S, \mathsf{pk}_S, x_S) \leftarrow Q_i[j]$
$(\mathsf{sk}_R, \mathsf{pk}_R, x_R) \leftarrow Q_k[\ell]$
$\sigma \leftarrow \mathsf{Sign}(\mathsf{mpk}, \mathsf{sk}_S, \mathsf{pk}_R, m)$
$b \leftarrow \mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}_S, \mathsf{pk}_R, m, \sigma)$
Return $b \neq F(x_S, x_R)$

$\underline{\mathcal{O}_{\mathsf{Gen}}(x):}$
$c \leftarrow c + 1$
$(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x)$
$Q_c \leftarrow [(\mathsf{sk}, \mathsf{pk}, x)]$
Return $(\mathsf{sk}, \mathsf{pk})$
$\underline{\mathcal{O}_{\mathsf{ReRand}}(j):}$
If $j > c$ return $\perp$
$(\mathsf{sk}, \mathsf{pk}, x) \leftarrow Q_j[|Q_j|]$
$(\mathsf{sk}', \mathsf{pk}') \leftarrow \mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})$
$Q_j \leftarrow Q_j || (\mathsf{sk}', \mathsf{pk}', x)$
Return $(\mathsf{sk}', \mathsf{pk}')$

Figure 15: Correctness Experiment of a ul-PCS scheme.

$\underline{\mathsf{Det}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A})}$
$(F, \mathsf{st}) \leftarrow \mathcal{A}_1(1^\lambda)$
$(\mathsf{msk}, \mathsf{mpk}) \leftarrow \mathsf{Setup}(1^\lambda, F)$
$c := 0$
$(i, j) \leftarrow \mathcal{A}_2^{\mathcal{O}_{\mathsf{Gen}}, \mathcal{O}_{\mathsf{ReRand}}}(\mathsf{st}, \mathsf{mpk})$
$(\mathsf{sk}^*, \mathsf{pk}^*) \leftarrow Q_i[j]$
$i^* \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}^*, (Q_1, \ldots, Q_c))$
Return $i^* \neq i$

$\underline{\mathcal{O}_{\mathsf{Gen}}(x):}$
$c \leftarrow c + 1$
$(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x)$
$Q_c \leftarrow [(\mathsf{sk}, \mathsf{pk})]$
Return $(\mathsf{sk}, \mathsf{pk})$
$\underline{\mathcal{O}_{\mathsf{ReRand}}(j):}$
If $j > c$, return $\perp$
$(\mathsf{sk}, \mathsf{pk}) \leftarrow Q_j[|Q_j|]$
$(\mathsf{sk}', \mathsf{pk}') \leftarrow \mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})$
$Q_j \leftarrow Q_j || (\mathsf{sk}', \mathsf{pk}')$
Return $(\mathsf{sk}', \mathsf{pk}')$

Figure 16: Detectability Experiment of a ul-PCS scheme.

for all algorithms $\mathcal{A}$, and computationally zero-knowledge, if $\mathsf{Adv}^{\mathsf{ZK}}_{\mathsf{NIZK},\mathcal{A},\mathcal{S}}(\lambda) \leq \mathsf{negl}(\lambda)$ for all PPT algorithms $\mathcal{A}$.

Besides zero-knowledge and soundness, we rely on the notion of extractability [24].

Definition A.17 (Extractability). Let NIZK = (Setup, Prove, Verify) be a NIZK proof system for a relation R and the corresponding language L, let Ext = $(\mathsf{Ext}_1, \mathsf{Ext}_2)$ be a pair of algorithms (the extractor). We define the extraction advantages of an adversary $\mathcal{A}$ as

$$\mathsf{Adv}^{\mathsf{CRS}}_{\mathsf{NIZK},\mathcal{A}} := |\Pr[\mathsf{CRS} \leftarrow \mathsf{Setup}(1^\lambda); 1 \leftarrow \mathcal{A}(\mathsf{CRS})] - \Pr[(\mathsf{CRS}, \mathsf{st}) \leftarrow \mathsf{Ext}_1(1^\lambda); 1 \leftarrow \mathcal{A}(\mathsf{CRS})]|,$$

and

$$\mathsf{Adv}^{\mathsf{Extract}}_{\mathsf{NIZK},\mathcal{A}}(\lambda) := \Pr\begin{bmatrix}(\mathsf{CRS}_{\mathsf{Ext}}, \mathsf{st}_{\mathsf{Ext}}) \leftarrow \mathsf{Ext}_1(1^\lambda)\\ (x, \pi) \leftarrow \mathcal{A}(\mathsf{CRS}_{\mathsf{Ext}})\\ \mathsf{Verify}(\mathsf{CRS}_{\mathsf{Ext}}, x, \pi) = 1 \wedge\\ R(x, \mathsf{Ext}_2(\mathsf{CRS}_{\mathsf{Ext}}, \mathsf{st}_{\mathsf{Ext}}, x, \pi)) = 0\end{bmatrix}$$

A NIZK proof system NIZK is called extractable, with respect to an extractor Ext = $(\mathsf{Ext}_1, \mathsf{Ext}_2)$, if $\mathsf{Adv}^{\mathsf{CRS}}_{\mathsf{NIZK},\mathcal{A}} \leq \mathsf{negl}(\lambda)$ and $\mathsf{Adv}^{\mathsf{Extract}}_{\mathsf{NIZK},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$. Additionally, we call an extractable non-interactive zero-knowledge proof a non-interactive zero-knowledge proof of knowledge (NIZKPoK).

B FORMAL DEFINITIONS FOR UL-PCS

B.1 Formal Correctness Definition

A ul-PCS scheme is called correct if for all efficient adversaries $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ in experiment CORR, specified in Figure 15, the probability $\Pr[\mathsf{CORR}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A}) = 1]$ is negligible in the security parameter.

B.2 Formal Detectability Definition

Let Detect be an algorithm that takes as input the master public key mpk, a candidate key pk^*, and a list consisting of sequences of key pairs $(Q_1, \ldots, Q_c)$, and outputs an index or $\perp$. A ULPCS scheme is said to have the detectability property if there is an efficiently computable algorithm Detect such that for all efficient adversaries $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ in experiment DTCT, specified in Figure 16, the probability $\Pr[\mathsf{DTCT}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A}) = 1]$ is negligible in the security parameter.

B.3 Formal Unforgeability Definition

Definition B.1 (Existential Unforgeability of a PCS Scheme). Let ULPCS = (Setup, KeyGen, Sign, Verify) be a ul-PCS scheme that satisfies the detectability property. We define the experiment $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}$ in Figure 2, where all oracles are defined as in Section 3.1. The advantage of an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ is defined by

$$\mathsf{Adv}^{\mathsf{EUF\text{-}CMA}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = \Pr[\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A}) = 1].$$

Such a ul-PCS scheme ULPCS is called *existential unforgeable under adaptive chosen message attacks* or *existential unforgeable* for short if for any polynomial-time adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, there exists a negligible function negl such that: $\mathsf{Adv}^{\mathsf{EUF\text{-}CMA}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$. We further call a ul-PCS scheme T_{Rand}-unforgeable if the number of key rerandomization queries q is less than T_{Rand}, i.e. $q < T_{\mathsf{Rand}}$.

B.4 Formal Attribute-hiding Definition

Definition B.2 (IND-Based Attribute Hiding). Let ULPCS = (Setup, KeyGen, Sign, Verify) be a ul-PCS scheme that satisfies the detectability property. For $\beta \in \{0, 1\}$, we define the experiment $\mathsf{AH}^{\mathsf{ULPCS}}_\beta$ in Figure 3, where all oracles are defined as in Section 3.1. The advantage of an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ is defined by

$$\mathsf{Adv}^{\mathsf{AH}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = |\Pr[\mathsf{AH}^{\mathsf{ULPCS}}_0(1^\lambda, \mathcal{A}) = 1] - \Pr[\mathsf{AH}^{\mathsf{ULPCS}}_1(1^\lambda, \mathcal{A}) = 1]|.$$

We call an adversary *valid* if all of the following hold with probability 1 over the randomness of the adversary and all involved algorithms, where $i_{\max}$ denotes an upper bound on the number of queries to $\mathsf{QKeyGenLR}_\beta$:

- for every $((i,j), \mathsf{pk}_i^j, \mathsf{sk}_i^j, x_{i,0}, x_{i,1}) \in \mathcal{QC}$ and for all $((k,\ell), \mathsf{pk}_k^\ell, \mathsf{sk}_k^\ell, x_{k,0}, x_{k,1}) \in \mathcal{QK}$ we have $x_{i,0} = x_{i,1} =: x_i$ and $F(x_i, x_{k,0}) = F(x_i, x_{k,1})$,
- and for all $((i,j), \mathsf{pk}_i^j, \mathsf{pk}, m, \sigma) \in \mathcal{QS}$, $R \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}, (\mathcal{QK}_1, \ldots, \mathcal{QK}_{i_{\max}}))$, and $((i,j), \mathsf{pk}_i, \mathsf{sk}_i, x_{i,0}, x_{i,1}) \in \mathcal{QK}$, we either have $R = \perp$ or otherwise $F(x_{i,0}, x_{R,0}) = F(x_{i,1}, x_{R,1})$ holds.

Such a ul-PCS scheme ULPCS is called *attribute hiding* if for any valid polynomial-time adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, there exists a negligible function negl such that: $\mathsf{Adv}^{\mathsf{AH}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$. We call a ul-PCS scheme T_{Rand}-attribute-hiding if the number of key rerandomization queries q is less than T_{Rand}, i.e. $q < T_{\mathsf{Rand}}$. Finally, we call a ul-PCS scheme outsider-attribute-hiding (outsider-AH) if the adversary does not have access to the corruption oracle.

Note that outsider security models e.g. an outsider attacker who is analyzing a transaction graph [23, 26].

B.5 Formal Unlinkability Definition

Definition B.3. Let ULPCS = (Setup, KeyGen, Sign, Verify) be a ul-PCS scheme that satisfies the detectability property. For $\beta \in \{0,1\}$, we define the experiment $\mathsf{Link}^{\mathsf{ULPCS}}_\beta$ in Figure 4, where all oracles are defined as in Section 3.1. The advantage of an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ is defined by

$$\mathsf{Adv}^{\mathsf{Link}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = |\Pr[\mathsf{Link}^{\mathsf{ULPCS}}_0(1^\lambda, \mathcal{A}) = 1] - \Pr[\mathsf{Link}^{\mathsf{ULPCS}}_1(1^\lambda, \mathcal{A}) = 1]|.$$

We call such an ul-PCS scheme ULPCS *unlinkable* if for any polynomial-time adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$, there exists a negligible function negl such that: $\mathsf{Adv}^{\mathsf{Link}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$.

We call a ul-PCS scheme T_{Rand}-unlinkable if the number of key rerandomization queries q is less than T_{Rand}, i.e. $q < T_{\mathsf{Rand}}$.

C NOTE ON MULTI-CHALLENGE UNLINKABILITY

The definition of multi-challenge unlinkability is almost the same as the definition of unlinkability with the only difference that, instead of submitting a single challenge query x, the adversary has access to a key generation oracle QKeyGen that it can query using multiple attributes to obtain multiple challenge public keys. Additionally, the adversary can query the rerandomization oracle using an index i to obtain a rerandomized key for the public key associated with the index i. If the adversary wants to obtain the corresponding secret key for a public key, it can query the corruption oracle QCor using the corresponding index i. The signing oracle in this case QSign takes the same inputs as in the unforgeability and the attribute-hiding game. More formally:

Definition C.1. Let ULPCS = (Setup, KeyGen, Sign, Verify) be a ul-PCS scheme that satisfies the detectability property. For $\beta \in \{0,1\}$, we define the experiment $\mathsf{MC\text{-}Link}^{\mathsf{ULPCS}}_\beta$ in Figure 17, where the rerandomization oracle is defined as:

$\mathsf{QRandKey}_\beta(\cdot)$**:** On input i, do the following: if $\beta = 0$ then set $(\mathsf{pk}', \mathsf{sk}') \leftarrow \mathsf{RandKey}(\mathsf{mpk}, \mathsf{sk})$, and if $\beta = 1$ set $(\mathsf{pk}', \mathsf{sk}') \leftarrow \mathsf{KeyGen}(\mathsf{msk}, x)$ where x is the attribute recorded for $\mathcal{QK}_i$, and sk is taken from the entry $((i,j), \mathsf{pk}, \mathsf{sk}, x)$ entry of $\mathcal{QK}$ with the highest j for the give i. Finally, add $((i, j+1), \mathsf{pk}', \mathsf{sk}')$ to $\mathcal{QK}$ and return pk'.

oracles are defined as in Appendix C.

The advantage of an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ is defined by

$$\mathsf{Adv}^{\mathsf{MC\text{-}Link}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = |\Pr[\mathsf{MC\text{-}Link}^{\mathsf{ULPCS}}_0(1^\lambda, \mathcal{A}) = 1] - \Pr[\mathsf{MC\text{-}Link}^{\mathsf{ULPCS}}_1(1^\lambda, \mathcal{A}) = 1]|.$$

An adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$ is called valid, if no index i is queried to *both* oracles $\mathsf{QRandKey}_\beta(i)$ and $\mathsf{QCor}(i)$.

We call such a ul-PCS scheme ULPCS *unlinkable* if for any polynomial-time adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$, there exists a negligible function negl such that: $\mathsf{Adv}^{\mathsf{MC\text{-}Link}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) \leq \mathsf{negl}(\lambda)$.

$\mathsf{MC\text{-}Link}^{\mathsf{ULPCS}}_\beta(1^\lambda, \mathcal{A})$

$(F, \mathsf{st}_1) \leftarrow \mathcal{A}_1(\lambda)$

$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{Setup}(1^\lambda, F)$

$\alpha \leftarrow \mathcal{A}_2^{\mathsf{QKeyGen}(\cdot),\mathsf{QRandKey}_\beta(\cdot),\mathsf{QCor}(\cdot),\mathsf{QSign}(\cdot,\cdot,\cdot)}(\mathsf{mpk}, \mathsf{st}_1)$

Output: α

Figure 17: Many-Challenges Unlinkability game of ULPCS**.**

It is straightforward to verify that the single-challenge implies the multi-challenge extension. Formally, this extension is formalized by introducing the oracles QKeyGenC and QSign_β and defining the multi-challenge version in Figure 17. In the game, we maintain an additional set $\mathcal{QCK}$ (initially empty):

Theorem C.2 (Link implies MC-Link). *Let* ULPCS *be* Link *secure, then* ULPCS *is also* MC-Link *secure.*

Proof (Sketch). This proof proceeds using a simple hybrid argument using the following game:

Game G_k**:** For the first k keys that are being queried to the rerandomization oracle $\mathsf{QRandKey}_\beta$, fresh keys are generated, whereas for the remaining keys all queries asked to $\mathsf{QRandKey}_\beta$ are answered using rerandomized keys.

Let Q be the number of overall key queries, then it holds that

$$\mathsf{MC\text{-}Link}_0 = G_0 \approx \cdots \approx G_Q = \mathsf{MC\text{-}Link}_1$$

To conclude the proof, it needs to be shown that $G_{k-1} \approx G_k$ for all $k \in [Q]$. This can be done using a reduction to the Link security game by forwarding the k'th challenge query to the underlying challenger of the Link game and then reply using the obtained key. The remaining keys are generated using key generation queries to the underlying challenger. The obtained secret keys can then be used to answer potential corruption queries of the adversary. To answer signing queries, they are also directly forwarded to the underlying challenger or generated using the known secret keys.

Therefore it follows that $G_{k-1} \approx G_k$ for all $k \in [Q]$, which proves the theorem. □

D FORMAL DESCRIPTION OF THE UL-PCS SCHEMES

D.1 Formal Description of the separable ul-PCS scheme

In this section, we formally define the languages of the ul-PCS scheme and describe it formally in Figure 18. The proof of this schemes proceeds analogous to the proof of Theorem 4.1 since the scheme is an optimized version of the generic scheme. We point out the few minor differences at the end of Appendices E.2 to E.4.

Language $\mathcal{L}_1$: A statement $x_{\text{st}} := (T_{\text{Rand}}, \text{ID}_{\text{ctr}}, \text{vk}_{\text{sig}}^{\text{ctr}}, \text{ct}_{\text{ctr}}, \text{vk}_{\text{sig}}^{A,R}, \text{pk}_{\text{PKE}}^{A})$ is in the language $\mathcal{L}_1$ if it holds for a witness $w_{\text{st}} := (\text{k}, \text{ctr}, \text{vk}_{\text{sig}}, \text{sk}_{\text{sig}}, m_x, \sigma_{\text{sig}}^1, \sigma_{\text{ctr}})$ that:

- $\text{ctr} < T_{\text{Rand}}$
- $\text{ct}_{\text{ctr}} = \text{PKE.Enc}(\text{pk}_{\text{PKE}}^{A}, m_x)$
- $\text{ID}_{\text{ctr}} = \text{PRF.Eval}(\text{k}, \text{ctr})$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}^{A,R}, (\text{k}, \text{vk}_{\text{sig}}, m_x), \sigma_{\text{sig}}^1) = 1$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}, (\text{vk}_{\text{sig}}^{\text{ctr}}, \text{ID}_{\text{ctr}}), \sigma_{\text{ctr}}) = 1$

Language $\mathcal{L}_2$: A statement $x_{\text{st}} := (\text{ID}_S, \text{ct}_R, \text{vk}_{\text{sig}}^{A,S}, \text{pk}_{\text{PKE}}^{A})$ is in the language $\mathcal{L}_2$ if it holds for a witness $w_{\text{st}} := (\text{k}, \text{ctr}, \text{sk}_{\text{PKE}}^{A}, \sigma_{\text{sig}}^2)$ that:

- $\text{PKE.Dec}(\text{sk}_{\text{PKE}}^{A}, \text{ct}_R) = 1$
- $\text{ID}_S = \text{PRF.Eval}(\text{k}, \text{ctr})$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}^{A,S}, (\text{k}, \text{sk}_{\text{PKE}}^{A}), \sigma_{\text{sig}}^2) = 1$

$\underline{\text{Setup}(1^\lambda, F)}$:
$\text{CRS}_{\text{Rand}} \leftarrow \text{NIZK}_{\mathcal{L}_1}.\text{Setup}(1^\lambda)$
$\text{CRS}_{\text{Sign}} \leftarrow \text{NIZK}_{\mathcal{L}_2}.\text{Setup}(1^\lambda)$
$(\text{vk}_{\text{sig}}^{A,S}, \text{sk}_{\text{sig}}^{A,S}) \leftarrow \text{DS.Setup}(1^\lambda)$
$(\text{vk}_{\text{sig}}^{A,R}, \text{sk}_{\text{sig}}^{A,R}) \leftarrow \text{DS.Setup}(1^\lambda)$
$(\text{pk}_{\text{PKE}}^{A}, \text{sk}_{\text{PKE}}^{A}) \leftarrow \text{PKE.Setup}(1^\lambda)$
$\text{mpk} := (T_{\text{Rand}}, F, \text{CRS}_{\text{Rand}}, \text{CRS}_{\text{Sign}}, \text{vk}_{\text{sig}}^{A,S}, \text{vk}_{\text{sig}}^{A,R}, \text{pk}_{\text{Enc}}^{A})$
$\text{msk} := (F, \text{sk}_{\text{sig}}^{A,S}, \text{sk}_{\text{sig}}^{A,R}, \text{sk}_{\text{PKE}}^{A})$
Return (mpk, msk)

$\underline{\text{KeyGen}(\text{msk}, x)}$:
Parse msk as defined above
$\text{k} \leftarrow \{0,1\}^\lambda$
$(\text{sk}_{\text{sig}}, \text{vk}_{\text{sig}}) \leftarrow \text{DS.Setup}(1^\lambda)$
$m_x := R(x)$
$\sigma_{\text{sig}}^1 \leftarrow \text{DS.Sign}(\text{sk}_{\text{sig}}^{A,R}, (\text{k}, \text{vk}_{\text{sig}}, m_x))$
If $S(x) = 1$:
  $\sigma_{\text{sig}}^2 \leftarrow \text{DS.Sign}(\text{sk}_{\text{sig}}^{A,S}, (\text{k}, \text{sk}_{\text{PKE}}^{A}))$
  $\text{usk} := (\text{k}, \text{vk}_{\text{sig}}, \text{sk}_{\text{sig}}, \sigma_{\text{sig}}^1, \sigma_{\text{sig}}^2, m_x, \text{sk}_{\text{PKE}}^{A})$
Else ($S(x) = 0$):
  $\text{usk} := (\text{k}, \text{vk}_{\text{sig}}, \text{sk}_{\text{sig}}, \sigma_{\text{sig}}^1, \sigma_{\text{sig}}^2 := \varepsilon, m_x, \text{sk}_{\text{PKE}}^{A} := \varepsilon)$
$(\text{pk}_0, \text{sk}_0) \leftarrow \text{RandKey}(\text{mpk}, (\text{usk}, -1, \perp))$
Return $(\text{pk}_0, \text{sk}_0)$

Figure 18a: The setup and key generation algorithm of our unlinkable PCS scheme for separable policies.

D.2 Formal Description of the Role-based ul-PCS scheme

In this section, we formally define the languages of the role-based ul-PCS scheme and describe it formally in Figure 19. The proof of this schemes proceeds analogous to the proof of Theorem 4.1 since the scheme is an optimized version of the generic scheme. We point out the few minor differences at the end of Appendices E.2 to E.4.

Language $\mathcal{L}_1$: A statement $x_{\text{st}} := (T_{\text{Rand}}, \text{ID}_{\text{ctr}}, \text{vk}_{\text{sig}}^{\text{ctr}}, \vec{M}' := (\text{A}_1', \text{A}_2', \text{G}_2'), \text{vk}_{\text{sig}}^{A})$ is in the language $\mathcal{L}_1$ if it holds for a witness $w_{\text{st}} := (\text{k}, \text{ctr}, \text{vk}_{\text{sig}}, \text{sk}_{\text{sig}}, w_{\text{k}}, \sigma_{\text{sig}}^1, \sigma_{\text{ctr}})$ that:

- $\text{ctr} < T_{\text{Rand}}$
- $\text{ACC.MemVrf}(\text{A}_1', \text{k}, w_{\text{k}}) = 1$ where pp' is defined as $(p, \text{G}_1, \text{G}_2', e)$ (that is, the same as pp but with the generator G_2' instead.)
- $\text{ID}_{\text{ctr}} = \text{PRF.Eval}(\text{k}, \text{ctr})$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}^{A}, (\text{k}, \text{vk}_{\text{sig}}), \sigma_{\text{sig}}^1) = 1$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}, (\text{vk}_{\text{sig}}^{\text{ctr}}, \text{ID}_{\text{ctr}}), \sigma_{\text{ctr}}) = 1$

Language $\mathcal{L}_2$: A statement $x_{\text{st}} := (\text{ID}_S, \text{ct}_R, \text{vk}_{\text{sig}}^{A}, \text{pp}', \text{A}')$ is in the language $\mathcal{L}_2$ if it holds for a witness $w_{\text{st}} := (\text{k}, \text{ctr}, x, w, \sigma_{\text{sig}}^2)$ that:

RandKey(mpk, sk):

Parse mpk, usk as defined above and sk = (usk, ctr, ·)

ctr := ctr + 1

If ctr ≥ T_{Rand}: return ⊥

$\text{ID}_{\text{ctr}} := \text{PRF.Eval}(k, \text{ctr})$

$(\text{vk}_{\text{sig}}^{\text{ctr}}, \text{sk}_{\text{sig}}^{\text{ctr}}) \leftarrow \text{DS.Setup}(1^\lambda)$

$\sigma_{\text{ctr}} \leftarrow \text{DS.Sign}(\text{sk}_{\text{sig}}, (\text{vk}_{\text{sig}}^{\text{ctr}}, \text{ID}_{\text{ctr}}))$

$\text{ct}_{\text{ctr}} \leftarrow \text{PKE.Enc}(\text{pk}_{\text{PKE}}^A, m_x)$

$\pi_{\text{ctr}} \leftarrow \text{NIZK}_{\mathcal{L}_1}.\text{Prove}(\text{CRS}_{\text{Rand}},$
$(T_{\text{Rand}}, \text{ID}_{\text{ctr}}, \text{vk}_{\text{sig}}^{\text{ctr}}, \text{ct}_{\text{ctr}}, \text{vk}_{\text{sig}}^{A,R}, \text{pk}_{\text{PKE}}^A), (\text{usk}, \sigma_{\text{ctr}}))$

$\text{pk}_{\text{ctr}} := (\text{ID}_{\text{ctr}}, \text{vk}_{\text{sig}}^{\text{ctr}}, \text{ct}_{\text{ctr}}, \pi_{\text{ctr}})$

Return $(\text{pk}_{\text{ctr}}, \text{sk}_{\text{ctr}} := (\text{usk}, \text{ctr}, \text{sk}_{\text{sig}}^{\text{ctr}}))$

Sign(mpk, sk, pk_R, m):

Parse mpk, sk := (usk, ctr, sk_{ctr}) and usk as above

If ValidPK(mpk, pk_R) = 0 : return ⊥

$\text{ID}_S := \text{PRF.Eval}(k, \text{ctr})$

If $\text{sk}_{\text{PKE}}^A = \varepsilon$: return ⊥

If $\text{PKE.Dec}(\text{sk}_{\text{PKE}}^A, \text{ct}_R) = 0$: return ⊥

$\pi_s \leftarrow \text{NIZK}_{\mathcal{L}_2}.\text{Prove}(\text{CRS}_{\text{Sign}}, (\text{ID}_S, \text{ct}_R, \text{vk}_{\text{sig}}^{A,S}, \text{pk}_{\text{PKE}}^A), \text{sk})$

$\sigma \leftarrow \text{DS.Sign}(\text{sk}_{\text{ctr}}, (m, \text{pk}_R, \pi_s))$

Return (π_s, σ)

Verify(mpk, pk_S, pk_R, m, σ):

Parse mpk as defined above and $\sigma = (\pi, \sigma')$

If ValidPK(mpk, pk_S) = 0 or ValidPK(mpk, pk_R) = 0

Return ⊥

Return $(\text{NIZK}_{\mathcal{L}_2}.\text{Verify}(\text{CRS}_{\text{Sign}}, (\text{pk}_S, \text{pk}_R), \pi)$
$\wedge\ \text{DS.Verify}(\text{vk}_S, (m, \text{pk}_R, \pi), \sigma'))$

Figure 18b: The randomization, signing and verification algorithms of our unlinkable PCS scheme for separable policies.

- $\text{ACC.MemVrf}(\text{A}', x, w) = 1$
- $\text{ID}_S = \text{PRF.Eval}(k, \text{ctr})$
- $\text{DS.Verify}(\text{vk}_{\text{sig}}^A, (k, w), \sigma_{\text{sig}}^2) = 1$

The helper function for this scheme, ValidPK(mpk, pk), is defined differently to the helper function of the previous schemes. In more detail, it verifies the $\text{NIZK}_{\mathcal{L}_1}$ proof as well as the SPS-EQ signature and outputs 1 only if both verifications are successful.

Setup(1^λ, F):

Let pp be a bilinear setup

$\text{CRS}_{\text{Rand}} \leftarrow \text{NIZK}_{\mathcal{L}_1}.\text{Setup}(1^\lambda)$

$\text{CRS}_{\text{Sign}} \leftarrow \text{NIZK}_{\mathcal{L}_2}.\text{Setup}(1^\lambda)$

$(\text{sk}_{\text{sig}}^A, \text{vk}_{\text{sig}}^A) \leftarrow \text{DS.Setup}(1^\lambda)$

$(\text{vk}_{\text{SEQ}}^A, \text{sk}_{\text{SEQ}}^A) \leftarrow \text{SEQ.KeyGen}_{\mathcal{R}}(\text{pp})$

Parse F as an RBAC matrix with n_R roles denoted by $1, \ldots, n_R$

For all $y \in [n_R]$:

$(\text{A}_y, \alpha_y) \leftarrow \text{ACC.Create}(\text{pp})$

$S_y \leftarrow \{i \in [n_R] : F(i, y) = 1\}; W_y \leftarrow ()$

For all $i \in S_y$:

$w_i \leftarrow \text{ACC.Add}(\text{pp}, \text{A}_y, \alpha_y, i)$

$W_y \leftarrow W_y || (i, w_i)$

$\text{CRS} := (\text{CRS}_{\text{Rand}}, \text{CRS}_{\text{Sign}})$

$\text{mpk} := (\text{pp}, T_{\text{Rand}}, F, \text{CRS}, \text{vk}_{\text{sig}}^A, \text{vk}_{\text{SEQ}}^A)$

$\text{msk} := (\text{pp}, F, \text{sk}_{\text{SEQ}}^A, (\text{A}_j, W_j)_{j=1}^{n_R})$

Return (mpk, msk)

KeyGen(msk, x):

Parse msk as defined above

$\text{k} \leftarrow \{0,1\}^\lambda$

$(\text{sk}_{\text{sig}}, \text{vk}_{\text{sig}}) \leftarrow \text{DS.Setup}(1^\lambda)$

$(\text{A}_{\text{k}}, \alpha_{\text{k}}) \leftarrow \text{ACC.Create}(\text{pp})$

$w_{\text{k}} \leftarrow \text{ACC.Add}(\text{A}_{\text{k}}, \alpha_{\text{k}}, \text{k})$

$\vec{M} := (\text{A}_{\text{k}}, \text{A}_x, \text{G}_2)$

$\sigma_{\text{SEQ}} \leftarrow \text{SEQ.Sign}_{\mathcal{R}}(\text{sk}_{\text{SEQ}}^A, \vec{M})$

$\sigma_{\text{sig}} \leftarrow \text{DS.Sign}(\text{sk}_{\text{sig}}^A, (\text{k}, \text{vk}_{\text{sig}}))$

$W := ()$

For each $y \in [n_R] : F(x, y) = 1$ do:

Retrieve $(x, w) \in W_y$

$W \leftarrow W || (w, \text{DS.Sign}(\text{sk}_{\text{sig}}^A, (\text{k}, w)))$

$\text{usk} = (\vec{M}, \sigma_{\text{SEQ}}, W, w_{\text{k}}, \text{k}, \text{vk}_{\text{sig}}, \text{sk}_{\text{sig}}, \sigma_{\text{sig}}, x)$

$(\text{pk}_0, \text{sk}_0) \leftarrow \text{RandKey}(\text{mpk}, (\text{usk}, -1, \bot))$

Return $(\text{pk}_0, \text{sk}_0)$

Figure 19a: The setup and key generation algorithm of our unlinkable PCS scheme for RBAC policies.

RandKey(mpk, sk):

Parse mpk, usk as defined above and sk = (usk, ctr, ·)

ctr := ctr + 1

If ctr ≥ T_{Rand}: return ⊥

$\mathsf{ID}_{\mathsf{ctr}} := \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$

$(\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{sk}^{\mathsf{ctr}}_{\mathsf{sig}}) \leftarrow \mathsf{DS.Setup}(1^\lambda)$

$\sigma_{\mathsf{ctr}} \leftarrow \mathsf{DS.Sign}(\mathsf{sk}_{\mathsf{sig}}, (\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{ID}_{\mathsf{ctr}}))$

$\mu_{\mathsf{ctr}} \leftarrow \mathbb{Z}^*_p$

$(\vec{M}', \sigma'_{\mathsf{SEQ}}) \leftarrow \mathsf{SEQ.ChgRep}_{\mathcal{R}}(\mathsf{vk}^A_{\mathsf{SEQ}}, \vec{M}, \sigma_{\mathsf{SEQ}}, \mu_{\mathsf{ctr}})$

$\pi_{\mathsf{ctr}} \leftarrow \mathsf{NIZK}_{\mathcal{L}_1}.\mathsf{Prove}(\mathsf{CRS}_{\mathsf{Rand}},$
$(T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \vec{M}', \mathsf{vk}^A_{\mathsf{sig}},), (\mathsf{usk}, \sigma_{\mathsf{ctr}}))$

$\mathsf{pk}_{\mathsf{ctr}} := (\mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \vec{M}', \sigma'_{\mathsf{SEQ}}, \pi_{\mathsf{ctr}})$

Return $(\mathsf{pk}_{\mathsf{ctr}}, \mathsf{sk}_{\mathsf{ctr}} := (\mathsf{usk}, \mathsf{ctr}, \mathsf{sk}^{\mathsf{ctr}}_{\mathsf{sig}}))$

Sign(mpk, sk, pk_R, m):

Parse mpk, sk := (usk, ctr, $\mathsf{sk}_{\mathsf{ctr}}$) and usk as above

If ValidPK(mpk, pk_R) = 0 : return ⊥

$\mathsf{ID}_S := \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$

Parse $\mathsf{pk}_R = (\ldots, (\mathsf{A}, \mathsf{A}', \mathsf{G}'_2), \ldots)$

Let $\mathsf{pp}' \leftarrow (p, \mathsf{G}_1, \mathsf{G}'_2, e)$

If $\nexists\, w^* \in W \mid \mathsf{ACC.MemVrf}(\mathsf{pp}', \mathsf{A}', x, w^*)$: return ⊥

Find $w^* \in W \mid \mathsf{ACC.MemVrf}(\mathsf{pp}', \mathsf{A}', x, w^*) = 1$

$\pi_s \leftarrow \mathsf{NIZK}_{\mathcal{L}_2}.\mathsf{Prove}(\mathsf{CRS}_{\mathsf{Sign}}, (\mathsf{ID}_S, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{pp}', \mathsf{A}'), \mathsf{sk})$

$\sigma \leftarrow \mathsf{DS.Sign}(\mathsf{sk}_{\mathsf{ctr}}, (m, \mathsf{pk}_R, \pi_s))$

Return (π_s, σ)

Verify(mpk, $\mathsf{pk}_S, \mathsf{pk}_R, m, \sigma$):

Parse mpk as defined above and $\sigma = (\pi, \sigma')$

If ValidPK(mpk, pk_S) = 0 or ValidPK(mpk, pk_R) = 0
Return ⊥

Return $(\mathsf{NIZK}_{\mathcal{L}_2}.\mathsf{Verify}(\mathsf{CRS}_{\mathsf{Sign}}, (\mathsf{pk}_S, \mathsf{pk}_R), \pi)$
$\wedge\ \mathsf{DS.Verify}(\mathsf{vk}_S, (m, \mathsf{pk}_R, \pi), \sigma'))$

Figure 19b: The rerandomization, signing and verification algorithm of our unlinkable PCS scheme for RBAC policies.

E SECURITY ANALYSIS

Here, we present the formal proof of the ul-PCS scheme for generic policies (Theorem 4.1). We prove three theorems in this supplement where each theorem covers one aspect, i.e., unforgeability, attribute-hiding, and unlinkability, respectively. Furthermore, we also argue the detactability of the schemes. For the sake of notation, we denote the unlinkable PCS scheme for generic policies $F(x, y)$ by ULPCS. The concrete specification as pseudo-code can be found in the submission.

The proofs of Theorems 4.2 and 4.3 for separable and role-based policies, respectively, are given by describing which arguments need to be adjusted to accommodate the replacement of the PE scheme in these constructions. We describe these adjustments for unforgeability, attribute-hiding, and unlinkability right after the proofs of the generic scheme.

E.1 Detectability

The detect algorithm Detect behaves the same in all of the three different schemes. It takes as an input the master public key mpk, the challenge public key pk^* as well as the lists $(Q_1, \ldots, Q_c)$. It then behaves as follows: for all $i \in [c]$, it generates the maximal amount of rerandomizations. In more detail, for all $i \in [c]$, it executes as many rerandomizations of the keys contained in $Q_i[]$ until $Q_i[]$ contains T_{Rand} keys. Afterwards, it searches all the lists $Q_i[]$ and if it finds an index pair (i, i') for which it holds that $Q_i[i'] = (\mathsf{pk}^*, \mathsf{sk}^*)$, then it adds i to its final list Q. After Det has iterated over all lists $(Q_1, \ldots, Q_c)$, we distinguish between three cases: first, Q only contains a single i, second, Q contains multiple i's and, third, Q is empty. In the first case, Det simply outputs the single i, in the second case, Det outputs the lower of the two indices contained in Q and, in the third case, Det outputs ⊥. To argue the correctness of Det, we need to analyze the three different cases. We start by analyzing the third case. The third case can never occur because the key pk^* is generated by checking $Q_i[j]$ and therefore the detect algorithm Det will also find this index pair. In the first case, Det behaves correct since there is only a single index pair which explains the key pk^* and this is output by Det. The second case, can only occur if a key collision has happened as defined in the event $\mathsf{KeyColl}_{\mathcal{A}}$ below, which is negligible due to the security of the PRF (see below for the argument). Therefore, it follows that the algorithm Det is correct with probability $1 - \mathsf{negl}(\lambda)$, which concludes the detectability argument.

E.2 Unforgeability

Theorem E.1. *Let* $T_{\mathsf{Rand}} = \mathsf{poly}(\lambda)$. *If* DS = (Setup, Sign, Verify) *is an* EUF-CMA*-secure signature scheme,* PRF *a secure pseudorandom function,* $\mathsf{NIZK}_{\mathcal{L}_1}$ = (Setup, Prove, Verify) *a knowledge sound proof system for language* $\mathcal{L}_1$ *and* $\mathsf{NIZK}_{\mathcal{L}_2}$ = (Setup, Prove, Verify) *is a knowledge sound proof system for language* $\mathcal{L}_2$*, then* ULPCS *described in Figure 5 is* T_{Rand} EUF-CMA *secure, i.e. it holds that* $\mathsf{Adv}^{\mathsf{EUF\text{-}CMA}}_{\mathsf{ULPCS},\mathcal{A}} = \mathsf{negl}(\lambda)$.

Proof. Consider the random experiment $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A})$ for which we define the following two events:

- Event $\mathsf{KeyColl}_{\mathcal{A}}$: The adversary $\mathcal{A}$ terminates and it holds that there are indices i, i', j, j' with $i \neq j$ or $i' \neq j'$ such that $((i, i'), \mathsf{pk}_i, \cdot, \cdot), ((j, j'), \mathsf{pk}_j, \cdot, \cdot) \in \mathcal{QK}$, where $\mathsf{pk}_i = (\mathsf{ID}_i, \ldots)$, $\mathsf{pk}_j = (\mathsf{ID}_j, \ldots)$, for which $\mathsf{ID}_i = \mathsf{ID}_j$.
- Event $\mathsf{KeyForge}_{\mathcal{A}}$: The adversary $\mathcal{A}$ terminates with output $(\mathsf{pk}_S, \mathsf{pk}_R, m, \sigma)$ and there exists an entry $(\cdot, \mathsf{pk}^*_S, \mathsf{pk}^*_R, m^*, \sigma^*) \in \mathcal{QS} \cup \{(\mathsf{pk}_S, \mathsf{pk}_R, m, \sigma)\}$ for which the following condition holds: $\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}^*_S, \mathsf{pk}^*_R, m^*, \sigma^*) = 1 \wedge (S = \bot \vee R = \bot)$ where $S \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}^*_S, (\mathcal{QK}_1, \ldots, \mathcal{QK}_{i_{\max}}))$ and $R \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}^*_R, (\mathcal{QK}_1, \ldots, \mathcal{QK}_{i_{\max}}))$.

We denote the winning condition of the experiment by the event $\mathsf{WIN}_{\mathcal{A}}$ and split it into two parts:

- Event $\mathsf{WIN1}_{\mathcal{A}}$: The adversary generates the output $(\mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*)$ for which it holds that $\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*) = 1 \wedge \exists (i,j), \mathsf{sk}, x\ \forall (i',j'), \sigma :\ ((i,j), \mathsf{pk}, \mathsf{sk}, x) \in \mathcal{QK} \setminus \mathcal{QC} \wedge ((i',j'), \mathsf{pk}, \mathsf{pk}^*, m^*, \sigma) \notin \mathcal{QS}$.
- Event $\mathsf{WIN2}_{\mathcal{A}}$: The adversary $\mathcal{A}$ generates the output $(\mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*)$ for which it holds that $\mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*) = 1 \wedge \left[(S \neq \perp) \wedge (R \neq \perp) \Rightarrow F(x_S, x_R) = 0\right]$ where $S \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}, (\mathcal{QK}_1, \ldots, \mathcal{QK}_{i_{\max}}))$, $R \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}^*, (\mathcal{QK}_1, \ldots, \mathcal{QK}_{i_{\max}}))$ and x_S and x_R denote the respective attributes.

By Lemma E.2 and Lemma E.3, we obtain

$$\Pr[\mathsf{KeyForge}_{\mathcal{A}}] = \mathsf{negl}(\lambda) \text{ and } \Pr[\mathsf{KeyColl}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$$

for adversaries $\mathcal{B}_1$ and $\mathcal{B}_2'$ which are constructed based on $\mathcal{A}$ and have roughly the same efficiency as $\mathcal{A}$.

Finally, we obtain by Lemma E.4 and by Lemma E.5 that

$$\Pr[\mathsf{WIN1}_{\mathcal{A}}] = \mathsf{negl}(\lambda) \text{ and}$$
$$\Pr[\mathsf{WIN2}_{\mathcal{A}} \cap \overline{\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}}] = \mathsf{negl}(\lambda).$$

By definition of the events, we have

$$\begin{aligned}\Pr[\mathsf{WIN}_{\mathcal{A}}] &\leq \Pr[\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}] \\ &\quad + \Pr[\mathsf{WIN}_{\mathcal{A}} \cap \overline{\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}}] \\ &\leq \Pr[\mathsf{KeyColl}_{\mathcal{A}}] + \Pr[\mathsf{KeyForge}_{\mathcal{A}}] \\ &\quad + \Pr[\mathsf{WIN1}_{\mathcal{A}} \cap \overline{\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}}] \\ &\quad + \Pr[\mathsf{WIN2}_{\mathcal{A}} \cap \overline{\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}}].\end{aligned}$$

This concludes the proof of the theorem. □

Lemma E.2. *It holds that* $\Pr[\mathsf{KeyColl}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$.

Proof. To bound the probability for the occurrence of $\mathsf{KeyColl}_{\mathcal{A}}$, we need to bound the probability that there exist two honestly generated/rerandomized keys $\mathsf{pk} := (\mathsf{ID}, \ldots)$ and $\mathsf{pk}' := (\mathsf{ID}', \ldots)$ with $\mathsf{ID} = \mathsf{ID}'$. The ID of an honestly generated key is generated using a PRF evaluation as well as an attached zero-knowledge proof that proves that the resulting string is indeed an honest PRF evaluation. By relying on the soundness of the zero-knowledge proof, it is ensured that the resulting ID is indeed a valid PRF evaluation, which, by the n-instance/parallel composable security of the PRF, allows us to consider the ID's in this analysis as randomly sampled. Therefore, to conclude the proof of the lemma, it suffices to bound the collision probability for randomly sampled identities.

In our setting, we have n different keys that are being generated, where each of those keys can be randomized T times. This means that overall $n \cdot T$ different ID's are being sampled. The probability that all of these ID's are different is $(1 - \frac{1}{2^\lambda}) \cdot (1 - \frac{2}{2^\lambda}) \cdot \ldots \cdot (1 - \frac{n \cdot T - 1}{2^\lambda}) \prod_{k=1}^{nT-1} (1 - \frac{k}{2^\lambda})$. For this probability it holds that $\prod_{k=1}^{nT-1} (1 - \frac{k}{2^\lambda}) \geq (1 - \frac{nT-1}{2^\lambda})^{nT-1}$, which, in turn, can be bounded using Bernoulli's inequality $(1 - \frac{nT-1}{2^\lambda})^{nT-1} \geq 1 - (nT-1) \cdot \frac{nT-1}{2^\lambda} = 1 - \frac{(nT-1)^2}{2^\lambda}$. Considering now the complementary event that at least one collision of ID's occurs, then the resulting probability for this event is equal to $1 - (1 - \frac{(nT-1)^2}{2^\lambda}) = \frac{(nT-1)^2}{2^\lambda}$, which is negligible in λ. This concludes the proof of the lemma. □

Lemma E.3. *Let* $\mathsf{DS} = (\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$ *be an* EUF-CMA*-secure signature scheme and* $\mathsf{NIZK}_{\mathcal{L}_1} = (\mathsf{Setup}, \mathsf{Prove}, \mathsf{Verify})$ *is a knowledge sound proof system for* $\mathcal{L}_1$*, then* $\Pr[\mathsf{KeyForge}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$.

Proof. On a high-level, the adversary needs to prove a wrong claim which can either be done by attacking the NIZK directly, or if the NIZK is extractable, then the attacker must attack the underlying signature scheme in order to possess a valid witness.

We first make a first transition to a hybrid world $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}_{\mathsf{Hyb}}$, which is identical to $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}$ except that we replace $\mathsf{NIZK}_{\mathcal{L}_1}.\mathsf{Setup}(1^\lambda)$ by the CRS simulation algorithm Ext_1 associated to the NIZK scheme which also outputs the state $\mathsf{st}_{\mathsf{Rand}}$ for the second extraction algorithm Ext_2. All above defined events are still defined in this hybrid experiment. It follows directly from the knowledge soundness property of the NIZK, using a standard reduction, that

$$\Pr[\mathsf{KeyForge}_{\mathcal{A}}] \leq \Pr_{\mathsf{Hyb}}\left[\mathsf{KeyForge}_{\mathcal{A}}\right] + \mathsf{negl}(\lambda),$$

where $\Pr_{\mathsf{Hyb}}[.]$ makes explicit that this probability is taken w.r.t. experiment $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}_{\mathsf{Hyb}}$.

Now, to bound the probability of the occurrence of KeyForge, we need to bound three different subcases:

(1) The adversary is not able to forge a signature σ^1_{sig} or σ^2_{sig} that would suffice as a proof for the relation $R_{\mathcal{L}_1}$.
(2) The adversary is not able to forge a signature σ_{k+1} that would suffice as a proof for the relation $R_{\mathcal{L}_1}$.
(3) The adversary is not able to break the soundness of the underlying $\mathsf{NIZK}_{\mathcal{L}_1}$ to generate a valid proof without being in possession of a witness.

To bound the first case above, we now build an adversary $\mathcal{B}$ that simulates $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}_{\mathsf{Hyb}}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}$ experiment. We show that if $\mathcal{A}$ outputs $(\mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*)$ as defined in event KeyForge, then it can be used as a forgeability attack in the $\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}$ experiment unless a certain failure event $\mathsf{Fail}_{\mathsf{ext}}$ occurs in the reduction, which we then relate to the extraction advantage.

The adversary $\mathcal{B}$ behaves using the algorithms described in the protocol with the only difference that it does not generate the key pair $(\mathsf{vk}^A_{\mathsf{sig}}, \mathsf{sk}^A_{\mathsf{sig}})$ on its own but obtains it from an underlying challenger. Also the corresponding signatures $\sigma^1_{\mathsf{sig}}, \sigma^2_{\mathsf{sig}}$ and σ^3_{sig}, that are the outputs of key generation queries, are not generated by $\mathcal{B}$ directly but through signing oracle queries of $\mathcal{B}$ to its underlying challenger.

When $\mathcal{A}$ terminates with $(\mathsf{pk}^*_S := (\mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \pi^*_S), \mathsf{pk}^*_R := (\mathsf{ID}^*_R, \mathsf{vk}^*_R, \mathsf{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$, $\mathcal{B}_1$ first checks whether the conditions of event $\mathsf{KeyForge}_{\mathcal{A}}$ holds, using the detect procedure which will output S' and R'. If the conditions of $\mathsf{KeyForge}_{\mathcal{A}}$ do not hold, then abort. For the remainder of the proof we assume that, WLOG the condition is fulfilled w.r.t. S'. The R' case follows accordingly.

If the conditions of event $\mathsf{KeyForge}_{\mathcal{A}}$ are fulfilled, then $\mathcal{B}$ calls $(\mathsf{usk}^*, \sigma^*) \leftarrow \mathsf{Ext}_2(\mathsf{CRS}_{\mathsf{Rand}}, \mathsf{st}_{\mathsf{Rand}}, (T_{\mathsf{Rand}}, \mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}}), \pi^*_S))$ and checks whether $(x := (T_{\mathsf{Rand}}, \mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \mathsf{vk}^A_{\mathsf{sig}},$

$\mathsf{mpk}_{\mathsf{PE}}), w := (\mathsf{usk}^*, \sigma^*)) \in R_{\mathcal{L}_1}$ (which is efficiently checkable). Afterwards, $\mathcal{B}$ parses $\mathsf{usk}^* := (\mathsf{k}^*, \mathsf{vk}^*_{\mathsf{sig}}, \mathsf{sk}^*_{\mathsf{sig}}, \sigma^{*,1}_{\mathsf{sig}}, \sigma^{*,2}_{\mathsf{sig}}, \sigma^{*,3}_{\mathsf{sig}}, x^*, \mathsf{sk}^*_{f_x})$ it checks if $\mathsf{DS.Verify}(\mathsf{vk}^A_{\mathsf{sig}}, (\mathsf{k}^*, x^*), \sigma^{*,1}_{\mathsf{sig}}) = 1$ or $\mathsf{DS.Verify}(\mathsf{vk}^A_{\mathsf{sig}}, (\mathsf{k}^*, \mathsf{vk}^*_{\mathsf{sig}}), \sigma^{*,2}_{\mathsf{sig}}) = 1$ and submits the corresponding message-signature-pair that verifies, i.e. either $((\mathsf{k}^*, x^*), \sigma^{*,1}_{\mathsf{sig}})$ or $((\mathsf{k}^*, \mathsf{vk}^*_{\mathsf{sig}}), \sigma^{*,2}_{\mathsf{sig}})$, to its challenger if it has not been previously output by the signing oracle. Otherwise, it aborts.

Before we analyze what happens in the case that $(x, w) \notin R_{\mathcal{L}_1}$, we need to bound the case where the adversary $\mathcal{A}$ outputs a forgery for the signature σ_{k+1}. This part of the proof, i.e. the adversary $\mathcal{B}$ in this case, almost behaves as before, with the only difference that the adversary $\mathcal{B}$ randomly samples a value $i \leftarrow [q]$, where q is the number of key generation queries asked by the adversary $\mathcal{A}$, receives $\mathsf{vk}_{\mathsf{sig}}$ from the underlying challenger and uses $\mathsf{vk}_{\mathsf{sig}}$ from the challenger to answer the i'th key generation query asked by $\mathcal{A}$. To finish the key generation and for further rerandomization queries that are asked for the i'th key, the adversary $\mathcal{B}$ uses the signing oracle of its underlying challenger. When $\mathcal{A}$ terminates with $(\mathsf{pk}^*_S := (\mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \pi^*_S), \mathsf{pk}^*_R := (\mathsf{ID}^*_R, \mathsf{vk}^*_R, \mathsf{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$, $\mathcal{B}_1$ first checks whether the conditions of event $\mathsf{KeyForge}_{\mathcal{A}}$ holds, using the detect procedure which will output S' and R'. If the conditions of $\mathsf{KeyForge}_{\mathcal{A}}$ do not hold, then it aborts. Also, as described above, we assume that, WLOG the condition is fulfilled w.r.t. S'. The R' case follows accordingly. If the conditions of event $\mathsf{KeyForge}_{\mathcal{A}}$ are fulfilled, then $\mathcal{B}$ calls $(\mathsf{usk}^*, \sigma^*) \leftarrow \mathsf{Ext}_2(\mathsf{CRS}_{\mathsf{Rand}}, \mathsf{st}_{\mathsf{Rand}}, (T_{\mathsf{Rand}}, \mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}}), \pi^*_S)$, checks whether $(x := (T_{\mathsf{Rand}}, \mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}}), w := (\mathsf{usk}^*, \sigma^*)) \in R_{\mathcal{L}_1}$ (which is efficiently checkable) and if S' identified by Detect corresponds to the key that has been generated as the answer to the i'th query. Afterwards, $\mathcal{B}$ checks if $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}, (\mathsf{pk}^*_S \| \mathsf{ID}^*_S), \sigma^*) = 1$ and submits the signature σ^*, if it passes the test and has not been previously output by the signing oracle of the underlying challenger, as a forgery. Otherwise, it aborts. To conclude the analysis we argue that the above described case occurs with probability $\frac{1}{q}$, which is exactly the probability that the adversary $\mathcal{B}$ guesses the index for the rerandomized key correctly.

If $(x, w) \notin R_{\mathcal{L}_1}$ then abort with failure event $\mathsf{Fail}_{\mathsf{ext}}$. Therefore, taking into account the two reductions described above, it holds that the advantage can be reduced to the unforgeability of the underlying signature scheme with probability $\mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{KeyForge}_{\mathcal{A}} \cap \overline{\mathsf{Fail}_{\mathsf{ext}}}\right]$. This, in turn, results in the fact that $\mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{KeyForge}_{\mathcal{A}}\right] = \mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{KeyForge}_{\mathcal{A}} \cap \overline{\mathsf{Fail}_{\mathsf{ext}}}\right] + \mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{Fail}_{\mathsf{ext}}\right] + \mathsf{negl}(\lambda)$.

Since a forgery for the underlying $\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}$ experiment only occurs with negligible probability, it follows that $\mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{KeyForge}_{\mathcal{A}} \cap \overline{\mathsf{Fail}_{\mathsf{ext}}}\right] = \mathsf{negl}(\lambda) + \frac{1}{q}\mathsf{negl}(\lambda) = \mathsf{negl}(\lambda)$ (after the two analysis above) and, to conclude the proof, it only remains to show that $\mathrm{Pr}_{\mathrm{Hyb}}\left[\mathsf{Fail}_{\mathsf{ext}}\right] = \mathsf{negl}(\lambda)$. This can be done by relying on the soundness property of the underlying $\mathsf{NIZK}_{\mathcal{L}_1}$ as mentioned in the second of the two cases above.

To conclude the proof, it remains to show that $\Pr[\mathsf{Fail}_{\mathsf{Ext}}] = \mathsf{negl}(\lambda)$. Also here, we assume that, WLOG the condition is fulfilled w.r.t. S'. The R' case follows accordingly. Our adversary $\mathcal{B}'$ for this case receives as an input the $\mathsf{CRS}_{\mathsf{Rand}}$ and executes the same instructions as $\mathcal{B}$, with the exceptions that it generates $(\mathsf{vk}^A_{\mathsf{sig}}, \mathsf{sk}^A_{\mathsf{sig}})$ by itself and uses it to generate the corresponding signatures by itself. Additionally, when $\mathcal{A}$ terminates with output $(\mathsf{pk}^*_S := (\mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \pi^*_S), \mathsf{pk}^*_R := (\mathsf{ID}^*_R, \mathsf{vk}^*_R, \mathsf{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$, $\mathcal{B}'$ behaves as $\mathcal{B}$ without running the extractor. Instead, it just outputs $(x := (T_{\mathsf{Rand}}, \mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \mathsf{vk}^A_{\mathsf{sig}}, \mathsf{mpk}_{\mathsf{PE}}), \pi^*_S)$ in case the conditions of $\mathsf{KeyForge}_{\mathcal{A}}$ are satisfied (note that the extractor is run as part of the knowledge soundness experiment). As above, the emulation towards $\mathcal{A}$ is perfect until the point where $\mathcal{B}'$ would abort. This results in the claimed advantage since the event of interest is that the extractor Ext_2 is called precisely on the accepting proof string π^*_S output by $\mathcal{A}$ which produces a witness w but for which $(x, w) \notin R_{\mathcal{L}_1}$. This concludes the proof of the lemma. □

Lemma E.4. *Let* $\mathsf{DS} = (\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$ *be an* EUF-CMA*-secure signature scheme, then* $\Pr[\mathsf{WIN1}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$.

Proof. To prove this lemma, we construct an adversary $\mathcal{B}$ that simulates $\mathsf{EUF\text{-}CMA}^{\mathsf{ULPCS}}$ towards $\mathcal{A}$. We show that if $\mathcal{A}$ outputs $(\mathsf{pk}, \mathsf{pk}^*, m^*, \sigma^*)$ as defined in event WIN1, then it can be used in a forgeability attack in the $\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}$ experiment.

Let q denote the number of queries to QRandKey. The adversary $\mathcal{B}$ behaves exactly as described in the experiment, with the only difference that it randomly samples values $i \leftarrow [q], j \leftarrow [\ell]$, where q denotes the number of queries to QKeyGen and ℓ denotes the number of queries to QRandKey, and, to reply to the j'th QRandKey query of the i'th key, it uses the key vk obtained from its underlying challenger. If later a signature query is being asked for the j'th rerandomization of the i'th key, then the adversary $\mathcal{B}$ relies on the signing oracle of its underlying challenger to generate the final signature. In case that the i'th key is being corrupted, the adversary $\mathcal{B}$ aborts.

Finally, when $\mathcal{A}$ terminates with output $(\mathsf{pk}^*_S := (\mathsf{ID}^*_S, \mathsf{vk}^*_S, \mathsf{ct}^*_S, \pi^*_S), \mathsf{pk}^*_R := (\mathsf{ID}^*_R, \mathsf{vk}^*_R, \mathsf{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$ check the conditions of WIN1 and check furthermore that the forgery output by $\mathcal{A}$ corresponds to the j'th rerandomization of the i'th key. If this is not the case, $\mathcal{B}$ aborts. If both of the conditions are satisfied, the adversary $\mathcal{B}$ outputs $((m^*, \mathsf{pk}^*_R, \pi^*), \sigma')$ as its forgery to the underlying $\mathsf{EUF\text{-}CMA}^{\mathsf{DS}}$ experiment.

To analyze the above reduction, we need to calculate the probability with which the adversary $\mathcal{B}$ succeeds with the advantage of $\mathcal{A}$. This happens with probability $\frac{1}{q\ell}$, since the adversary $\mathcal{B}$ needs to guess the correct key i that is used by the adversary $\mathcal{A}$ in the forgery, as well as the correct rerandomization j. Since $q\ell$ is polynomial in the security parameter, the lemma follows. □

Lemma E.5. *Let* $\mathsf{DS} = (\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$ *be an* EUF-CMA*-secure signature scheme and* $\mathsf{NIZK}_{\mathcal{L}_2} = (\mathsf{Setup}, \mathsf{Prove}, \mathsf{Verify})$ *is a knowledge sound proof system for* $\mathcal{L}_2$*, then* $\Pr[\mathsf{WIN2}_{\mathcal{A}} \cap \overline{\mathsf{KeyColl}_{\mathcal{A}} \cup \mathsf{KeyForge}_{\mathcal{A}}}] = \mathsf{negl}(\lambda)$.

Proof. On a high-level, in this setting, the adversary needs to prove a wrong claim which can either be done by attacking the NIZK

directly, or if the NIZK is extractable, then the attacker must attack the underlying signature scheme in order to possess a valid witness.

We first make a first transition to a hybrid world $\text{EUF-CMA}^{\text{ULPCS}}_{\text{Hyb}}$, which is identical to $\text{EUF-CMA}^{\text{ULPCS}}$ except that we replace $\text{NIZK}_{\mathcal{L}_2}.\text{Setup}(1^\lambda)$ by the CRS simulation algorithm Ext_1 associated to the NIZK scheme which also outputs the state st_{Sign} for the second extraction algorithm Ext_2. It follows directly from the knowledge soundness property of the NIZK, using a standard reduction, that

$$\Pr[\text{WIN2}_{\mathcal{A}} \cap \overline{\text{KeyColl}_{\mathcal{A}} \cup \text{KeyForge}_{\mathcal{A}}}] \leq \Pr_{\text{Hyb}}\left[\text{WIN2}_{\mathcal{A}} \cap \overline{\text{KeyColl}_{\mathcal{A}} \cup \text{KeyForge}_{\mathcal{A}}}\right] + \text{negl}(\lambda),$$

where $\Pr_{\text{Hyb}}[.]$ makes explicit that this probability is taken w.r.t. the experiment $\text{EUF-CMA}^{\text{ULPCS}}_{\text{Hyb}}$.

Now, to bound the probability of the occurrence of KeyForge, we need to bound two different subcases:

(1) The adversary is not able to forge a signature σ^3_{sig} that would suffice as a proof for the relation $R_{\mathcal{L}_2}$.
(2) The adversary is not able to break the soundness of the underlying $\text{NIZK}_{\mathcal{L}_2}$ to generate a valid proof without being in possession of a witness.

To bound the first case above, we build an adversary $\mathcal{B}$ that simulates $\text{EUF-CMA}^{\text{ULPCS}}_{\text{Hyb}}$ towards $\mathcal{A}$ when interacting with the underlying $\text{EUF-CMA}^{\text{DS}}$ experiment. We show that if $\mathcal{A}$ outputs $(\text{pk}, \text{pk}^*, m^*, \sigma^*)$ as defined in event WIN2, then it can be used as a forgeability attack in the $\text{EUF-CMA}^{\text{DS}}$ experiment unless a certain failure event Fail_{ext} occurs in the reduction, which we can then relate to the extraction advantage.

The adversary $\mathcal{B}$ behaves using as described in the protocol with the only difference that it does not generate the key pair $(\text{vk}^A_{\text{sig}}, \text{sk}^A_{\text{sig}})$ on its own but obtains it from an underlying challenger. Also the corresponding signatures $\sigma^1_{\text{sig}}, \sigma^2_{\text{sig}}$ and σ^3_{sig}, that are the outputs of key generation queries, are not generated by $\mathcal{B}$ directly but through signing oracle queries of $\mathcal{B}$ to its underlying challenger.

When $\mathcal{A}$ terminates with $(\text{pk}^*_S := (\text{ID}^*_S, \text{vk}^*_S, \text{ct}^*_S, \pi^*_S), \text{pk}^*_R := (\text{ID}^*_R, \text{vk}^*_R, \text{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$, $\mathcal{B}_1$ first checks whether the conditions of event WIN2 are fulfilled (and $\text{KeyForge}_{\mathcal{A}}$ and $\text{KeyColl}_{\mathcal{A}}$ did not occur), using the detect procedure which will output S' and R'. If the conditions of WIN2 are not fulfilled, then $\mathcal{B}$ aborts.

If the conditions of event WIN2, and not $\text{KeyForge}_{\mathcal{A}}$ and $\text{KeyColl}_{\mathcal{A}}$, are fulfilled, then $\mathcal{B}$ calls $\text{sk}^* \leftarrow \text{Ext}_2(\text{CRS}_{\text{Sign}}, \text{st}_{\text{Sign}}, (\text{ID}^*_S, \text{vk}^A_{\text{sig}}, \text{ct}^*_R), \pi^*)$ and checks whether $(x := (\text{ID}^*_S, \text{ct}^*_R, \text{vk}^A_{\text{sig}}), w := \text{sk}^*) \in R_{\mathcal{L}_2}$ (which is efficiently checkable). Afterwards, $\mathcal{B}$ parses $\text{sk}^* := (\text{usk}^*, \text{ctr}^*, \text{sk}^{*,\text{ctr}}_{\text{sig}})$ and $\text{usk}^* := (\text{k}^*, \text{vk}^*_{\text{sig}}, \text{sk}^*_{\text{sig}}, \sigma^{*,1}_{\text{sig}}, \sigma^{*,2}_{\text{sig}}, \sigma^{*,3}_{\text{sig}}, x^*, \text{sk}^*_{f_x})$, checks if $\text{DS.Verify}(\text{vk}^A_{\text{sig}}, (\text{k}^*, \text{sk}^*_{f_x}), \sigma^{*,2}_{\text{sig}}) = 1$ and submits the message-signature-pair $((\text{k}^*, \text{sk}^*_{f_x}), \sigma^{*,3}_{\text{sig}})$ to its challenger if it has not been previously output by the signing oracle. Otherwise, it aborts.

If $(x, w) \notin R_{\mathcal{L}_2}$ then abort with failure event Fail_{ext}. Therefore, since the described reduction is perfect, it holds that the advantage can be reduced to the unforgeability of the underlying signature scheme with probability $\Pr_{\text{Hyb}}\left[\text{KeyForge}_{\mathcal{A}} \cap \overline{\text{Fail}_{\text{ext}}}\right]$. This, in turn, results in the fact that $\Pr_{\text{Hyb}}\left[\text{KeyForge}_{\mathcal{A}}\right] = \Pr_{\text{Hyb}}\left[\text{KeyForge}_{\mathcal{A}} \cap \overline{\text{Fail}_{\text{ext}}}\right] + \Pr_{\text{Hyb}}\left[\text{Fail}_{\text{ext}}\right] + \text{negl}(\lambda)$.

Since a forgery for the underlying $\text{EUF-CMA}^{\text{DS}}$ experiment only occurs with negligible probability, it follows that $\Pr_{\text{Hyb}}\left[\text{KeyForge}_{\mathcal{A}} \cap \overline{\text{Fail}_{\text{ext}}}\right] = \text{negl}(\lambda)$ and, to conclude the proof, it only remains to show that $\Pr_{\text{Hyb}}[\text{Fail}_{\text{ext}}] = \text{negl}(\lambda)$. This can be done by relying on the soundness property of the underlying $\text{NIZK}_{\mathcal{L}_2}$ as mentioned in the second of the two cases above.

Our adversary $\mathcal{B}'$ in the case that Fail_{Ext} occurs receives as an input the CRS_{Sign} and executes the same instructions as $\mathcal{B}$, with the exceptions that it generates $(\text{vk}^A_{\text{sig}}, \text{sk}^A_{\text{sig}})$ by itself and can use it to generate signatures by itself. In addition, when $\mathcal{A}$ terminates with output $(\text{pk}^*_S := (\text{ID}^*_S, \text{vk}^*_S, \text{ct}^*_S, \pi^*_S), \text{pk}^*_R := (\text{ID}^*_R, \text{vk}^*_R, \text{ct}^*_R, \pi^*_R), m^*, \sigma^* := (\pi^*, \sigma'))$, $\mathcal{B}'$ behaves as $\mathcal{B}$ but does not execute the final steps running the extractor, but instead just outputs $(x := (\text{ID}^*_S, \text{ct}^*_R, \text{vk}^A_{\text{sig}}), \pi^*)$ in case the conditions of WIN2, and not $\text{KeyForge}_{\mathcal{A}}$ and $\text{KeyColl}_{\mathcal{A}}$, are satisfied (note that the extractor is run as part of the knowledge soundness experiment). As above, the emulation towards $\mathcal{A}$ is perfect until the point where $\mathcal{B}'$ would abort. This results in the claimed advantage since the event of interest is that the extractor Ext_2 is called precisely on the accepting proof string π^* output by $\mathcal{A}$ which produces a witness w such that $(x, w) \notin R_{\mathcal{L}_2}$. This concludes the proof of the lemma. □

E.2.1 Analysis in the case of Separable & RBAC Policies.

Separable Policies. The security proofs for the scheme covering separable policies proceeds exactly in the same way as the proof described above, i.e. in the proof of Lemmas E.3 and E.5, where the occurence of exactly the same subevents are being bounded. The reason is that we still have the same components, signatures and encryptions, but thanks to the pre-computation of $S(x)$ and $R(x)$ we can mimic the PE part of the generic scheme accurately and securely.

RBAC Policies. The proof for the scheme covering RBAC policies has a few differences when bounding the event $\text{KeyForge}_{\mathcal{A}}$ (Lemma E.3). Instead of bounding the unforgeability of the signatures σ^1_{sig} and σ^2_{sig} for the PE-based scheme, in the RBAC scheme it is necessary to bound the unforgeability of σ^1_{sig} and invoke the unforgeability of the SEQ scheme to make sure that none of the parties can obtain a different role (akin to re-encryptions of attributes of the generic scheme). This was previously captured within the NIZK, and now, thanks to SEQ, can be verified outside the NIZK. To argue unforgeability now, we first rely on the secure adaptation property of SEQ to argue that the signature generated using $\text{ChgRep}_{\mathcal{R}}$ is indistinguishable from a signature generated using Sign. Afterwards, we can conclude the proof by relying on the unforgeability of the SEQ scheme and the fact that with overwhelming probability, every party is its own equivalence class, which stems from the fact that for each party, the first component of the vector $\vec{M}$ is a randomly

sampled group element. For the proof of event WIN2 (Lemma E.5), we also need to rely on the weak soundness property of the accumulator to argue that an adversary cannot forge a signature by forging a valid accumulator. To rely on the accumulator soundness, we observe that a party cannot claim to own different roles than the ones it got issued (akin to the signature on the attribute x in the generic scheme).

E.3 Attribute Hiding

In this section, we prove the attribute hiding of our scheme.

Theorem E.6. *Let* $T_{\mathsf{Rand}} = \mathsf{poly}(\lambda)$. *If* $\mathsf{PE} = (\mathsf{PE.Setup}, \mathsf{PE.KeyGen}, \mathsf{PE.Enc}, \mathsf{PE.Dec})$ *is a predicate encryption scheme,* $\mathsf{NIZK}_{\mathcal{L}_1} = (\mathsf{NIZK.Setup}, \mathsf{NIZK.Prove}, \mathsf{NIZK.Verify})$ *is a NIZK proof system for language* $\mathcal{L}_1$, $\mathsf{NIZK}_{\mathcal{L}_2} = (\mathsf{NIZK.Setup}, \mathsf{NIZK.Prove}, \mathsf{NIZK.Verify})$ *is a NIZK proof system for language* $\mathcal{L}_2$ *and* $\mathsf{DS} = (\mathsf{DS.Setup}, \mathsf{DS.Sign}, \mathsf{DS.Verify})$ *an unforgeable signature scheme, then the construction* $\mathsf{ULPCS} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$, *defined in Figures 12 and 13, is attribute hiding. Namely, for any valid PPT adversary* $\mathcal{A}$, *it holds that* $\mathsf{Adv}^{\mathsf{AH}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = \mathsf{negl}(\lambda)$.

Proof. To prove this statement, we use a hybrid argument where the games are defined as follows:

Game G_0**:** This game is defined as $\mathsf{AH}^{\mathsf{ULPCS}}_0(1^\lambda, \mathcal{A})$.

Game G_1**:** In this game, we change the behavior of the sign oracle QSign and define a modified sign oracle QSign′. The oracle QSign′ is defined as QSign with the difference that it only answers queries for receiver keys that have been honestly generated (keys that have been output by the key generation oracle $\mathsf{QKeyGenLR}_0$ or are an honest rerandomization of these keys, which can be determined using the Detect procedure), for a query (i, pk', m) with $(i, \cdot, \cdot, \cdot, \cdot) \notin \mathcal{QK}$ or $(j, \cdot, \cdot, \cdot, \cdot) \notin \mathcal{QK}$, where $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$ the sign oracle QSign′ outputs $\perp$. The transition from G_0 to G_1 is justified by the bounds on the key forgery event as described in the proof of Theorem E.1. We show this transition more formally in Lemma E.7.

Game G_2**:** In this game, we change from an honestly generated $\mathsf{CRS}_{\mathsf{Rand}}$ and honestly generated proofs to a simulated $\mathsf{CRS}_{\mathsf{Rand}}$ and simulated proofs. That is, for the randomization of challenge keys that can never be corrupted, i.e. for the challenge query (x_0, x_1) it holds that $x_0 \neq x_1$, the proof in the randomization for $R_{\mathcal{L}_1}$ is simulated and therefore does not require the attributes used in the witness. Furthermore, we also remove the signatures σ^1_{sig} and σ^2_{sig} from the scheme in this transition. The transition from G_1 to G_2 is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_1}$. We show this transition more formally in Lemma E.8.

Game G_3**:** In this game, we change from an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs to a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs. That is, upon a signing query we check, from the transcript of the generated keys and using the detect function, if the requested key pair in the signing query fulfills the policy. If this is the case, the proof π_s is simulated using $\mathsf{CRS}_{\mathsf{Sign}}$. Here, we furthermore also remove the key sk_{f_x} as well as the signature σ^3_{sig} from the key generation procedure. As in the previous transition, this also only happens for explicitly honest keys, i.e. keys where $x_0 \neq x_1$. The transition from G_2 to G_3 is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_2}$. We show this transition more formally in Lemma E.9.

Game G_4**:** In this game, we change the attributes used in the rerandomization for the explicitly honest challenge keys from x_0 to x_1 for all i by changing the encryption that is being generated in the randomization procedure. The transition from G_3 to G_4 is justified by the attribute-hiding property of PE. We show this transition more formally in Lemma E.10.

Game G_5**:** In this game, we change back from a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs to an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs. Here, we also reintroduce the signature σ^3_{sig} but this time w.r.t. the challenge messages x_1. Similar to the transition from G_2 to G_3, this transition is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_2}$.

Game G_6**:** In this game, we change back from a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs to an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs. Here, we also reintroduce the signatures σ^1_{sig} and σ^2_{sig} but this time w.r.t. the challenge messages x_1. Similar to the transition from G_1 to G_2, this transition is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_1}$.

Game G_7**:** This game is the $\mathsf{AH}^{\mathsf{ULPCS}}_1(1^\lambda, \mathcal{A})$ game. In this game, we change the behavior of the signing oracle back from QSign′ to QSign. Similar to the transition from G_0 to G_1, this transition is justified by the event $\mathsf{KeyForge}_{\mathcal{A}}$.

From the definition of the games it is clear that

$$\mathsf{AH}^{\mathsf{ULPCS}}_0 = G_0 \approx G_1 \approx \cdots \approx G_7 = \mathsf{AH}^{\mathsf{ULPCS}}_1$$

and hence the theorem follows. □

Lemma E.7 (Transition from G_0 to G_1). *The games* G_0 *and* G_1 *are computationally indistinguishable.*

Proof (Sketch). As described above, the difference between the games G_0 and G_1 is that in the game G_0 the adversary $\mathcal{A}$ has access to the sign oracle QSign and in the game G_1 the adversary $\mathcal{A}$ has access to the sign oracle QSign′, which we informally described above and which is formally defined as:

QSign′(i, pk', m)**:** On input a (sender) index i, a (receiver) public key pk', and a message m, if $\mathcal{QK}$ contains an entry $(i, \mathsf{pk}, \mathsf{sk}, x_0, x_1) \in \mathcal{QK}$ and an entry $(j, \mathsf{pk}', \mathsf{sk}', x'_0, x'_1) \in \mathcal{QK}$ with $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$, then return $\sigma \leftarrow \mathsf{ULPCS.Sign}(\mathsf{mpk}, \mathsf{sk}, \mathsf{pk}', m)$ and add $(i, \mathsf{pk}, \mathsf{pk}', m, \sigma)$ to $\mathcal{QS}$. Otherwise, return $\perp$.

Compared to the oracle QSign′, the signing oracle QSign does not require the receiver key pk' to have been previously output by the challenger or being a rerandomization of a key output by the challenger, i.e. $(j, \cdot, \cdot, \cdot, \cdot) \notin \mathcal{QK}$ with $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$, to obtain as a reply a valid signature $\sigma \neq \perp$. This is not possible for the oracle QSign′ where every query using a receiver key pk' that has not been generated by the challenger or rerandomized from a challenger key, i.e. $(j, \cdot, \cdot, \cdot, \cdot) \notin \mathcal{QK}$ with $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$, results in an invalid signature $\sigma = \perp$.

Therefore, to show that the games G_0 and G_1 are indistinguishable, it suffices to show that the probability that the adversary queries the signing oracle QSign using a receiver key pk' that has not been previously generated by the challenger or rerandomized from a challenger key, i.e. $(j, \cdot, \cdot, \cdot, \cdot) \notin \mathcal{QK}$ with $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$, and that leads to a valid signature $\sigma \neq \perp$ is negligible. We denote this as the event $\mathsf{SignForge}_{\mathcal{A}}$.

For the event $\mathsf{SignForge}_{\mathcal{A}}$ to occur, the adversary $\mathcal{A}$ needs to generate a receiver key that has a valid zero-knowledge proof π, or where the underlying witness is forged, a valid signature $\mathsf{sig}^1_{\mathsf{sig}}$ the signature scheme DS, i.e., it needs to generate a key $\mathsf{pk}' := (\mathsf{ID}', \mathsf{vk}', c', \pi)$ such that $\mathsf{NIZK}_{\mathcal{L}_1}.\mathsf{Verify}(\mathsf{CRS}_{\mathsf{Rand}}, (\mathsf{ID}', \mathsf{vk}', c', \mathsf{vk}^A_{\mathsf{sig}}), \pi) = 1$ where π is generated using $\mathsf{usk}' := (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{PE}}, \sigma^1_{\mathsf{sig}}, \sigma^2_{\mathsf{sig}}, \sigma^3_{\mathsf{sig}}, x)$ and it must holds that $\mathsf{DS.Verify}(\mathsf{vk}^A_{\mathsf{sig}}, (\mathsf{k}, x), \sigma^1_{\mathsf{sig}}) = 1$ and $\mathsf{DS.Verify}(\mathsf{vk}^A_{\mathsf{sig}}, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}), \sigma^2_{\mathsf{sig}}) = 1$. This means that adversary $\mathcal{A}$ must either break the soundness of NIZK or generate a key forgery for DS as captured by the event $\mathsf{KeyForge}_{\mathcal{A}}$ in the proof of Theorem E.1, and which can be defined and analyzed analogously here.

Therefore, the event $\mathsf{SignForge}_{\mathcal{A}}$ is bounded by KeyForge, i.e. $\Pr[\mathsf{SignForge}_{\mathcal{A}}] \leq \Pr[\mathsf{KeyForge}_{\mathcal{A}}]$, and the analysis of event $\mathsf{KeyForge}_{\mathcal{A}}$ follows the same reasoning as in Lemma E.3. This results in the fact that $\Pr[\mathsf{SignForge}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$, which proves the lemma. □

Lemma E.8 (Transition from G_1 to G_2). *The games G_1 and G_2 are computationally indistinguishable.*

Proof. We build an adversary $\mathcal{B}$ that simulates $G_{1+\beta}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{ZK}^{\mathsf{NIZK}}_{\beta}$ experiment.

The adversary $\mathcal{B}$ behaves in the same way as described in G_1 with the difference that it does not generate $\mathsf{CRS}_{\mathsf{Rand}}$ by itself but receives it from the underlying challenger. Additionally, whenever the adversary $\mathcal{A}$ asks a rerandomization query to QRandKey for a key that cannot be corrupted, i.e. where the key generation query is for $x_0 \neq x_1$, the adversary $\mathcal{B}$ behaves as described in the protocol but uses the proof oracle of the challenger for the generation of the proof π_{k+1}. Furthermore, the signature σ^1_{sig} and σ^2_{sig} are not generated.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we argue that our emulation is perfect. The fact that the simulation is perfect follows since $\mathcal{B}$ generates all components of the statement for which the proof oracle is queried honestly.

In the case that the challenger outputs an honestly generated $\mathsf{CRS}_{\mathsf{Rand}}$ and honestly generated proofs, the adversary $\mathcal{B}$ is simulating the game G_1 and in the case that the challenger simulates the $\mathsf{CRS}_{\mathsf{Rand}}$ and the proofs, the adversary $\mathcal{B}$ is simulating the game G_2.

This concludes the simulation of the game $G_{1+\beta}$ and the lemma follows. □

Lemma E.9 (Transition from G_2 to G_3). *The games G_2 and G_3 are computationally indistinguishable.*

Proof. We build an adversary $\mathcal{B}$ that simulates $G_{2+\beta}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{ZK}^{\mathsf{NIZK}}_{\beta}$ experiment.

The adversary $\mathcal{B}$ behaves in the same way as described in G_2 with the difference that it does not generate $\mathsf{CRS}_{\mathsf{Sign}}$ by itself but receives it from the underlying challenger. Additionally, whenever the adversary $\mathcal{A}$ asks a signing query (i, pk', m) to QSign′, the adversary $\mathcal{B}$ computes $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$ and checks that $F(x_0, y_0) = 1$ where $(i, \cdot, \cdot, x_0, x_1) \in \mathcal{QK}$ and $(j, \cdot, \cdot, y_0, y_1) \in \mathcal{QK}$. If the check succeeds, then $\mathcal{B}$ queries its underlying proof oracle to obtain π_s and finishes the signature generation. Furthermore, for all keys that cannot be corrupted, i.e. where the key generation query is for $x_0 \neq x_1$, the signature σ^3_{sig} is not generated and the key sk_{f_x} is not generated. This makes the secret key completely independent of the attributes x_0/x_1.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we argue that our emulation is perfect. The fact that the simulation is perfect follows since $\mathcal{B}$ only submits proof queries to the underlying challenger for which the statement fulfills the relation $R_{\mathcal{L}_2}$, which $\mathcal{B}$ checks as described above as well as from the perfect correctness of the predicate encryption scheme. In more detail, by the perfect correctness of the predicate encryption scheme, we know that the challenger always replies, i.e., we have that $\mathsf{PE.Dec}(\mathsf{sk}_{f_x}, \mathsf{ct}_R) = F(x, y)$. Therefore, whenever a proof is simulated this matches the correct generation of a proof π_s.

In the case that the challenger outputs an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs, the adversary $\mathcal{B}$ is simulating the game G_2 and in the case that the challenger simulates the $\mathsf{CRS}_{\mathsf{Sign}}$ and the proofs, the adversary $\mathcal{B}$ is simulating the game G_3.

This concludes the simulation of the game $G_{2+\beta}$ and the lemma follows. □

Lemma E.10 (Transition from G_3 to G_4). *The games G_3 and G_4 are computationally indistinguishable.*

Proof. We build an adversary $\mathcal{B}$ that simulates $G_{3+\beta}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{AH}^{\mathsf{PE}}_{\beta}$ experiment.

The adversary $\mathcal{B}$ behaves in the same way as described in G_3 with the difference that whenever the adversary $\mathcal{A}$ asks a key generation query for a key that can be corrupted, i.e. $x := x_0 = x_1$, the adversary $\mathcal{B}$ asks its underlying key generation oracle using x to obtain sk_{f_x}.

Additionally, when $\mathcal{A}$ asks a rerandomization query to QRandKey for a key that cannot be corrupted, i.e. where the key generation query is for $x_0 \neq x_1$, the adversary $\mathcal{B}$ behaves as described in the protocol but uses its underlying left-or-right oracle for the generation of the ciphertext, i.e. for every rerandomization query for a key i, $\mathcal{B}$ retrieves $(i, \cdot, \cdot, x_0, x_1) \in \mathcal{QK}$ and submits (x_0, x_1) to its underlying challenger to obtain ct which it uses for the rerandomization.

Furthermore, for every sign query (j, pk_R, m) to QSign′ asked by $\mathcal{A}$, $\mathcal{B}$ computes $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$ checks the list $\mathcal{QK}$ to find $(i, \cdot, \cdot, x_0, x_1)$ and $(j, \cdot, \cdot, y_0, y_1)$. If no such entries exists, $\mathcal{B}$ outputs $\perp$. Otherwise, $\mathcal{B}$ checks that that the attributes associated with the public keys pk_S and pk_R fulfill the policy, i.e. it checks that $F(x^0, y^0) = 1$ and $F(x^1, y^1) = 1$, and if this is the case simulates the proof and generates the signature.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

In the next step, we need to argue that the adversary $\mathcal{B}$ is a valid adversary with respect to the $\mathsf{AH}^{\mathsf{PE}}_{\beta}$ experiment if the adversary $\mathcal{A}$ fulfills all the checks described above, i.e. is a valid adversary in the $G_{3+\beta}$ ($\mathsf{AH}^{\mathsf{ULPCS}}_{\beta}$) game. One of the validity requirements above (and in the attribute hiding game) that $\mathcal{A}$ needs to fulfill is that for every x where $x := x_0 = x_1$ with $(\cdot,\cdot,\cdot,x_0,x_1) \in \mathcal{QS}$ it needs to hold that $F(x,x_0) = F(x,x_1)$ for all the challenge queries (x_0,x_1). This results in the fact that $f_x(x_0) = f_x(x_1)$ for all $(\cdot,\cdot,\cdot,x,x) \in \mathcal{QC}$ and for all challenge queries (x_0,x_1). This matches exactly the validity requirements asked for $\mathcal{B}_2$ in the $\mathsf{AH}^{\mathsf{PE}}_{\beta}$ experiment. Therefore, it follows that the adversary $\mathcal{B}_2$ is a valid adversary with respect to the $\mathsf{AH}^{\mathsf{PE}}_{\beta}$ experiment and does not abort if the adversary $\mathcal{A}$ is a valid adversary in the game $G_{2+\beta}$ ($\mathsf{AH}^{\mathsf{ULPCS}}_{\beta}$).

To conclude the proof, we observe that the difference in the two games is the generation of the challenge rerandomization keys, which either consists of a ciphertext encrypting the attribute set x_0 or the attribute set x_1. The computation of the ciphertexts is done by the underlying challenger of the attribute-hiding game. Together with the analysis above, it follows that, for a valid adversary $\mathcal{A}$, the game $G_{3+\beta}$ is simulated towards $\mathcal{A}$ when the challenger encrypts the attribute set x_β for $\beta \in \{0,1\}$.

This concludes the simulation of the game $G_{3+\beta}$ and the lemma follows. □

E.3.1 Analysis in the case of Separable & RBAC Policies.

Separable Policies. The security proof for the scheme covering separable policies proceeds in almost the same way as the proof for general policies. The only difference is the transition from G_3 to G_4 (Lemma E.10), where in the proof for separable policies we need to rely on the IND-CPA security of the underlying public-key encryption scheme PKE instead of the attribute-hiding security of a PKE scheme.

RBAC Policies. For the security proof of the scheme covering RBAC policies, we also need to adjust the transition from game G_3 to G_4 (Lemma E.10). In this case, we need to rely on the class-hiding property as well as the secure adaptation property. In more detail, the class-hiding property guarantees that a switch from attributes x_0 to x_1 (in the case of outsider attribute-hiding or in the case of the equality policy) is possible and the secure adaptation property of SEQ ensures that the $\mathsf{ChgRep}_{\mathcal{R}}$ algorithm is as good as re-generating $\vec{M}$, which fulfills the same purpose as the re-encryption for the schemes covering general and separable policies.

E.4 Unlinkability

This section, covers the unlinkability proof of our schemes.

Theorem E.11. *Let* $T_{\mathsf{Rand}} = \mathsf{poly}(\lambda)$. *If* PRF *is a pseudorandom function,* $\mathsf{NIZK}_{\mathcal{L}_1} = (\mathsf{NIZK.Setup}, \mathsf{NIZK.Prove}, \mathsf{NIZK.Verify})$ *a NIZK proof system for* $\mathcal{L}_1$, $\mathsf{NIZK}_{\mathcal{L}_1} = (\mathsf{NIZK.Setup}, \mathsf{NIZK.Prove}, \mathsf{NIZK.Verify})$ *a NIZK proof system for* $\mathcal{L}_1$ *and* $\mathsf{DS} = (\mathsf{DS.Setup}, \mathsf{DS.Sign}, \mathsf{DS.Verify})$ *an unforgeable signature scheme, then the construction* $\mathsf{ULPCS} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$*, defined in Figures 12 and 13, is unlinkable. Namely, for any valid PPT adversary* $\mathcal{A}$*, it holds that* $\mathsf{Adv}^{\mathsf{Link}}_{\mathsf{ULPCS},\mathcal{A}}(\lambda) = \mathsf{negl}(\lambda)$.

Proof. To prove this statement, we use a hybrid argument where the games are defined as follows:

Game G_0**:** This game is the same as the experiment $\mathsf{Link}^{\mathsf{ULPCS}}_0(1^\lambda, \mathcal{A})$.

Game G_1**:** In this game, we change the behavior of the key generation oracle QKeyGen and define a modified key generation oracle QKeyGen′. The oracle QKeyGen′ is defined as QKeyGen with the difference that it does not output a key collision, i.e. it does not output the same public key twice, and therefore does also not output the same public key as the challenge key. More formally, if for a query x', the output is $\mathsf{pk}' := (\mathsf{ID}', \dots)$ where $\mathcal{QK}$ already contains $(\dots, \mathsf{pk}^* := (\mathsf{ID}^*, \dots), \dots)$ with $\mathsf{ID}' = \mathsf{ID}^*$ or $\mathsf{ID}' = \mathsf{ID}'$ with $\mathsf{pk} := (\mathsf{ID}, \dots)$ being the challenge public key, then the key-generation oracle QKeyGen′ outptus $\perp$, otherwise it returns pk'. The transition from G_0 to G_1 is justified by the bounds on the key collision event as described in the proof of Theorem E.1. We show this transition more formally in Lemma E.12.

Game G_2**:** In this game, we change the behavior of the sign oracle QSign and define a modified sign oracle QSign′. As in the proof of Theorem E.6, the oracle QSign′ is defined as QSign with the difference that it only answers queries for receiver keys that it can detect to have come out of the key-gen oracle. For further details on QSign′, we refer to the proof of Theorem E.6. The transition from G_1 to G_2 is justified by the bounds on the key forgery event as described in the proof of Theorem E.1 and because the detect property is fulfilled by the scheme. This transition has been shown in Lemma E.7.

Game G_3**:** In this game, we change from an honestly generated $\mathsf{CRS}_{\mathsf{Rand}}$ and honestly generated proofs w.r.t. rerandomizations of the challenge public key pk to a simulated $\mathsf{CRS}_{\mathsf{Rand}}$ and simulated proofs for the rerandomizations of pk. Due to the fact that the proofs for the rerandomizations are now simulated, the PRF key k is not needed as part of the witness anymore. The transition from G_2 to G_3 is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_1}$. We show this transition more formally in Lemma E.13.

Game G_4**:** In this game, we change from an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs for signing queries w.r.t. the challenge public key pk acting as the sender to a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs. That is, upon a signing query for pk, acting as the sender, we check, from the transcript of the generated keys and using the detect function, if the requested key pair in the signing query fulfills the policy. If this is the case, the proof π_s is simulated using $\mathsf{CRS}_{\mathsf{Sign}}$. Since the proof π_s is now simulated, the PRF key k is not needed as part of the witness anymore. The transition from G_3 to G_4 is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_2}$. We show this transition more formally in Lemma E.14.

Game G_5**:** In this game, we change from PRF evaluations for the updated ID's in the rerandomization of the challenge key pk to randomly sampled ID's. The transition from G_4 to G_5 is justified by the security of the PRF. We show this transition more formally in Lemma E.15.

Game G_6 : In this game, we change from randomly sampled updated ID's in the rerandomization of the challenge key pk to PRF evaluations w.r.t. different keys. In more detail, whenever a new rerandomization for the challenge key pk is generated a new PRF key k_i is sampled and the ID is generated by evaluation PRF using k_i on 0. The transition from G_5 to G_6 is justified by relying on the security of the PRF q-times where q is the number of rereandomization queries. Since it holds that $q < T_{\mathsf{Rand}}$, we can upper bound it by relying on the security of the PRF T_{Rand} times. We show this transition more formally in Lemma E.16.

Game G_7**:** In this game, we change back from a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs to an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs. Here, we also reintroduce the usage of the PRF key k, which is different for every rerandomization, into the generation of the proof. Similar to the transition from G_3 to G_4, this transition is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_2}$.

Game G_8**:** In this game, we change back from a simulated $\mathsf{CRS}_{\mathsf{Sign}}$ and simulated proofs to an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs. Here, we also reintroduce the usage of the PRF key k, which is different for every ID, into the generation of the proof. Similar to the transition from G_2 to G_3, this transition is justified by the zero-knowledge property of $\mathsf{NIZK}_{\mathcal{L}_1}$.

Game G_9**:** In this game, we change the behavior of the signing oracle back from QSign' to QSign. Similar to the transition from G_1 to G_2, this transition is justified by the event $\mathsf{KeyForge}_{\mathcal{A}}$.

Game G_{10}**:** This game is the $\mathsf{Link}_1^{\mathsf{ULPCS}}(1^\lambda, \mathcal{A})$ game. In this game, we change the behavior of the key generation oracle back from $\mathsf{QKeyGen}'$ to QKeyGen. Similar to the transition from G_0 to G_1, this transition is justified by the event $\mathsf{KeyColl}_{\mathcal{A}}$.

From the definition of the games it is clear that

$$\mathsf{Link}_0^{\mathsf{ULPCS}} = G_0 \approx G_1 \approx \cdots \approx G_{10} = \mathsf{Link}_1^{\mathsf{ULPCS}}$$

and hence the theorem follows. □

Lemma E.12 (Transition from G_0 to G_1). *The games G_0 and G_1 are computationally indistinguishable.*

Proof (Sketch). As described above, the difference between the games G_0 and G_1 is that in the game G_0 the adversar $\mathcal{A}$ has access to the key generation oracle QKeyGen and in the game G_1 the adversary $\mathcal{A}$ has access to the key generation oracle $\mathsf{QKeyGen}'$, which we informally described above and which is formally defined as:

$\mathsf{QKeyGen}'(x')$**:** On input an attribute set x', generate $\mathsf{pk}' := (\mathsf{ID}', \dots)$ and if $\mathcal{QK}$ already contains an entry $(\dots, \mathsf{pk}^* := (\mathsf{ID}^*, \dots), \dots)$ with $\mathsf{ID}' = \mathsf{ID}^*$ or if $\mathsf{ID}' = \mathsf{ID}$ where $\mathsf{pk} := (\mathsf{ID}, \dots)$ is the challenge public key, then output $\perp$. Otherwise, return pk'.

Compared to the oracle $\mathsf{QKeyGen}'$, the key generation oracle QKeyGen does not require a generated public pk' to be entirely new, i.e. $(\dots, \mathsf{pk}^* := (\mathsf{ID}^*, \dots), \dots) \notin \mathcal{QK}$ with $\mathsf{ID} \neq \mathsf{ID}^*$ and $\mathsf{ID}' \neq \mathsf{ID}$ where $\mathsf{pk} := (\mathsf{ID}, \dots)$ is the challenge public key. To show that the games G_0 and G_1 are indistinguishable, it suffices to show that the probability that two honestly generated IDs do not collide is negligible. This directly matches the description of the event $\mathsf{KeyColl}_{\mathcal{A}}$ defined in the proof of Theorem E.1 and, since the it holds that $\Pr[\mathsf{KeyColl}_{\mathcal{A}}] = \mathsf{negl}(\lambda)$, the lemma follows. □

Lemma E.13 (Transition from G_2 to G_3). *The games G_2 and G_3 are computationally indistinguishable.*

Proof. This proof is very similar to the proof of Lemma E.8.

We build an adversary $\mathcal{B}$ that simulates $G_{2+\beta}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{ZK}_\beta^{\mathsf{NIZK}}$ experiment.

The adversary $\mathcal{B}$ behaves in the same way as described in G_2 with the difference that it does not generate $\mathsf{CRS}_{\mathsf{Rand}}$ by itself but receives it from the underlying challenger. Additionally, whenever the adversary $\mathcal{A}$ asks a rerandomization query to QRandKey for the challenge public key pk, or a rerandomization of it, the adversary $\mathcal{B}$ behaves as described in the protocol but uses the proof oracle of the challenger for the generation of the proof π_{k+1}. Furthermore, the PRF key k is not used as a witness for the proof generation anymore.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we argue that our emulation is perfect. The fact that the simulation is perfect follows since $\mathcal{B}$ generates all components of the statement for which the proof oracle is queried honestly.

In the case that the challenger outputs an honestly generated $\mathsf{CRS}_{\mathsf{Rand}}$ and honestly generated proofs, the adversary $\mathcal{B}$ is simulating the game G_2 and in the case that the challenger simulates the $\mathsf{CRS}_{\mathsf{Rand}}$ and the proofs, the adversary $\mathcal{B}$ is simulating the game G_3.

This covers the simulation of the game $G_{2+\beta}$ and leads to the advantage mentioned in the lemma. □

Lemma E.14 (Transition from G_3 to G_4). *The games G_3 and G_4 are computationally indistinguishable.*

Proof. This proof is very similar to the proof of Lemma E.9.

We build an adversary $\mathcal{B}$ that simulates $G_{3+\beta}$ towards $\mathcal{A}$ when interacting with the underlying $\mathsf{ZK}_\beta^{\mathsf{NIZK}}$ experiment.

The adversary $\mathcal{B}$ behaves in the same way as described in G_3 with the difference that it does not generate $\mathsf{CRS}_{\mathsf{Sign}}$ by itself but receives it from the underlying challenger. Additionally, whenever the adversary $\mathcal{A}$ asks a signing query (pk', m) to QSign', the adversary $\mathcal{B}$ computes $j \leftarrow \mathsf{Detect}(\mathsf{mpk}, \mathsf{pk}', \mathcal{QK})$ and checks that $F(x, y) = 1$ where x is the challenge attribute and $(j, \cdot, \cdot, y) \in \mathcal{QK}$. If the check succeeds, then $\mathcal{B}$ queries its underlying proof oracle to obtain π_s and finishes the signature generation. Furthermore, the PRF key k is not needed for the generation of the proof π_s.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we argue that our emulation is perfect. The fact that the simulation is perfect follows since $\mathcal{B}$ only submits proof queries to the underlying challenger for which the statement fulfills the relation $R_{\mathcal{L}_2}$, which $\mathcal{B}$ checks as described above as well as from the perfect correctness of the predicate encryption scheme. In more detail, by the perfect correctness of the predicate encryption scheme, we know that the challenger always replies, i.e., we have that $\mathsf{PE.Dec}(\mathsf{sk}_{f_x}, \mathsf{ct}_R) = F(x, y)$. Therefore, whenever a proof is simulated this matches the correct generation of a proof π_s.

In the case that the challenger outputs an honestly generated $\mathsf{CRS}_{\mathsf{Sign}}$ and honestly generated proofs, the adversary $\mathcal{B}$ is simulating the game G_3 and in the case that the challenger simulates the $\mathsf{CRS}_{\mathsf{Sign}}$ and the proofs, the adversary $\mathcal{B}$ is simulating the game G_4.

This covers the simulation of the game $G_{3+\beta}$ and leads to the advantage mentioned in the lemma. □

Lemma E.15 (Transition from G_4 to G_5). *The games G_4 and G_5 are computationally indistinguishable.*

Proof. We build an adversary $\mathcal{B}$ that simulates $G_{4+\beta}$ towards $\mathcal{A}$ when interacting with the underlying security experiment for PRF.

The adversary $\mathcal{B}$ behaves in the same way as described in G_4 with the difference that whenever the adversary $\mathcal{A}$ asks the challenge key generation query or a rereandomization query, the adversary $\mathcal{B}$ submits the corresponding index, i.e. $i := 0$ for a key generation query and $i := i + 1$ for a rerandomizaton query, to the underlying PRF challenger and receives as a reply the ID that it uses to answer the queries.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we observe that the difference in the two games is the generation of the ID's, which is either PRF evaluation, in which case the simulation corresponds to game G_4, or a random value, in which case the simulation corresponds to game G_5.

This concludes the simulation of the game $G_{4+\beta}$ and the lemma follows. □

Lemma E.16 (Transition from G_5 to G_6). *The games G_5 and G_6 are computationally indistinguishable.*

Proof. We build an adversary $\mathcal{B}$ that simulates $G_{5+\beta}$ towards $\mathcal{A}$ when interacting with T instances[7] of the security experiment for PRF.

The adversary $\mathcal{B}$ behaves in the same way as described in G_5 with the difference that whenever the adversary $\mathcal{A}$ asks the challenge key generation query or a rereandomization query, the adversary $\mathcal{B}$ submits the 0 query to the i'th PRF instance. In more detail, for the challenge key generation query, the adversary $\mathcal{B}$ queries the first instance of the PRF experiment on 0 ot obtain the ID, for the first rerandomization query, the adversary $\mathcal{B}$ queries the second instance of the PRF experiment on 0 to obtain the ID and so on.

Finally, the adversary $\mathcal{B}$ outputs the same bit β' returned by $\mathcal{A}$.

To conclude the proof, we observe that the difference in the two games is the generation of the ID's, which is either a random value, in which case the simulation corresponds to game G_4, or a fresh PRF evaluation on zero, in which case the simulation corresponds to game G_5.

This concludes the simulation of the game $G_{4+\beta}$ and the lemma follows. □

E.4.1 Analysis in the case of Separable & RBAC Policies.

[7] where T instances means multiple PRF experiments that either all output random values or all PRF evaluations on a fresh key

Separable Policies. The security proof for the scheme covering separable policies proceeds in the same way as the proof for general policies. In both of these cases, general policies and separable policies, it is not necessary to rely on any of the properties of the predicate encryption scheme or the public-key encryption scheme since, in both cases, rerandomization and key generation, a fresh ciphertext is being generated. Therefore no indistinguishability needs to be argued.

RBAC Policies. In the RBAC case, there is a difference between the rerandomization and the key generation w.r.t. the SEQ scheme. In the case of a fresh key generation, a signature is generated on which $\mathsf{ChgRep}_{\mathcal{R}}$ is applied once. In the case of an actual rerandomization, the $\mathsf{ChgRep}_{\mathcal{R}}$ is applied multiple times on a single signature. To argue that these two cases are indistinguishable, we need to rely on the secure adaptation property of SEQ which can be understood as an additional game transition between G_5 and G_6 (Lemma E.16).

F DETAILS ON THE INSTANTIATIONS

F.1 Cryptographic Algorithms

Given the formal definitions from Appendix A, this section provides a detailed overview on the underlying cryptographic tools that we implement to realize ul-PCS.

F.1.1 Dodis-Yampolskiy PRF. This PRF is defined over a cyclic group $\mathbb{G}$ of prime order p with generator G and can be described as follows:

- $\mathsf{PRF.Eval}(\mathsf{k}, x)$: It takes a key $\mathsf{k} \in \mathbb{Z}_p$ and input x as inputs. It then computes $y = \mathrm{G}^{1/(\mathsf{k}+x)}$ and returns y as output.

In this scheme, pseudo-randomness is achieved through decisional Diffie-Hellman inversion assumption, which only holds for small domains, i.e. input x should be super-logarithmic in the security parameters.

F.1.2 BLS Signatures. For a given assymetric bilinear pairing group $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, \mathrm{G}_1, \mathrm{G}_2)$ and a hash-to-curve function $H : \{0,1\}^* \to \mathbb{G}_2$, as denoted by public parameters pp, we recall the BLS signatures [16] as follows:

- $\mathsf{DS.Setup}(1^\lambda)$: Take pp as input. Sample $x \xleftarrow{\$} \mathbb{Z}_p^*$. Return $(\mathsf{sk}, \mathsf{vk}) = (x, \mathrm{G}_1^x)$.
- $\mathsf{DS.Sign}(\mathsf{sk}, m)$: Take secret key sk and message $m \in \{0,1\}^*$ as inputs. Return $\sigma := H(m)^{\mathsf{sk}}$ as output.
- $\mathsf{DS.Verify}(\mathsf{vk}, \sigma, m)$: Take the verification key vk, a signature σ and message m as inputs. If the equation $e(\mathrm{G}_1, \sigma) = e(\mathsf{vk}, H(m))$ holds, return 1 and 0 otherwise.

F.1.3 FHS SPS-EQ. We recall the SPS-EQ construction proposed by Fuchsbauer et al. in [39] as follows:

- $\mathsf{SEQ.Setup}_{\mathcal{R}}(1^\lambda)$: Run $\mathsf{BG} \leftarrow \mathcal{BG}(1^\lambda)$ and return $\mathsf{pp} := \mathsf{BG}$ as output.
- $\mathsf{SEQ.KeyGen}_{\mathcal{R}}(\mathsf{pp}, \ell)$: Take pp and vector size $\ell > 1$ as inputs. Sample the secret key sk as a set of random integers $\mathsf{sk} := \{x_i\}_{i\in[1,\ell]} \xleftarrow{\$} (\mathbb{Z}_p^*)^\ell$. Compute $\mathsf{vk} := \{\hat{X}_i = \mathrm{G}_1^{x_i}\}_{i\in[1,\ell]}$. Return $(\mathsf{sk}, \mathsf{vk})$ as output.

- SEQ.Sign$_{\mathcal{R}}$(pp, sk, $\vec{M}$): Parse $\vec{M} := (M_i)_{i\in[1,\ell]} \in (\mathbb{G}_2)^\ell$ and sk : $\{x_i\}_{i\in[1,\ell]}$. Sample $a \xleftarrow{\$} \mathbb{Z}_p^*$ and return $\sigma := (R, S, T) := \left(\left(\prod_{i\in[1,\ell]} M_i^{x_i}\right)^a, \mathrm{G}_2^{1/a}, \mathrm{G}_1^{1/a}\right) \in \mathbb{G}_2^2 \times \mathbb{G}_1$ as output.
- SEQ.Verify$_{\mathcal{R}}$(pp, vk, $\vec{M}, \sigma$): Parse vk := $\{\hat{X}_i\}_{i\in[1,\ell]}$, $\vec{M} := (M_i)_{i\in[1,\ell]}$ and $\sigma := (R, S, T)$. If the equations $\prod_{i\in[1,\ell]} e(\hat{X}_i, M_i) = e(T, R)$ and $e(\mathrm{G}_1, S) = e(T, \mathrm{G}_2)$ hold and $M_i \neq 1_{\mathbb{G}_2}$ for $i \in [1, \ell]$ return 1 and 0 otherwise.
- SEQ.ChgRep$_{\mathcal{R}}$(pp, $\vec{M}, \sigma, \mu$, vk): Parse $\sigma := (R, S, T)$, $\vec{M} := (M_i)_{i\in[1,\ell]} \in (\mathbb{G}_2)^\ell$ and vk := $\{\hat{X}_i\}_{i\in[1,\ell]}$ along with an integer $\mu \in \mathbb{Z}_p^*$ as input. If the signature be valid it samples $\zeta \xleftarrow{\$} \mathbb{Z}_p^*$ and then returns $\sigma' := (R', S', T') \leftarrow (R^{\zeta\mu}, S^{1/\zeta}, T^{1/\zeta})$ on a re-randomized message $\vec{M}' = \vec{M}^\mu$ as output.

For simplicity we take a slightly modified variant of the described SPS-EQ as a standard SPS. Consider pairing group of the form $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, \mathrm{G}_1, \mathrm{G}_2)$, this scheme can be summarized as follows:

- SPS.KeyGen(pp, ℓ): Take pp and vector size $\ell > 1$ as inputs. Sample the secret key sk as a set of random integers sk := $\{x_i\}_{i\in[1,\ell]} \xleftarrow{\$} (\mathbb{Z}_p^*)^\ell$. Compute vk := $\{\hat{X}_i = \mathrm{G}_2^{x_i}\}_{i\in[1,\ell]}$. Return (sk, vk) as output.
- SPS.Sign(pp, sk, $\vec{M}$): Parse $\vec{M} := (M_i)_{i\in[1,\ell]} \in \mathbb{G}_1^\ell$ and sk := $\{x_i\}_{i\in[1,\ell]}$. Sample $a \xleftarrow{\$} \mathbb{Z}_p^*$ and output $\sigma := (R, S, T) := \left(\left(\prod_{i\in[1,\ell]} M_i^{x_i}\right)^a, \mathrm{G}_1^{1/a}, \mathrm{G}_2^{1/a}\right) \in \mathbb{G}_1^2 \times \mathbb{G}_2$.
- SPS.Verify(pp, vk, $\sigma, \vec{M}$): Parse vk := $\{\hat{X}_i\}_{i\in[1,\ell]} \in \mathbb{G}_2^\ell$, $\vec{M} := (M_i)_{i\in[1,\ell]} \in (\mathbb{G}_1)^\ell$ and $\sigma := (R, S, T)$. If both equations $\prod_{i\in[1,\ell]} e(M_i, \hat{X}_i) = e(R, T)$ and $e(S, \mathrm{G}_2) = e(\mathrm{G}_1, T)$ hold and $M_i \neq 1_{\mathbb{G}_1}$ for $i \in [1, \ell]$ return 1 and 0 otherwise.

F.1.4 ElGamal Encryption. Consider a group description $(\mathbb{G}, \mathrm{G}, p)$, the ElGamal encryption [32] can be formalized as follows:

- PKE.Setup(1^λ): It takes security parameter λ as input and then samples random integer sk $\leftarrow \mathbb{Z}_p^*$ and computes pk = G^{sk}. It then returns the key-pair (sk, pk) as output.
- PKE.Enc(pp, pk, m): It takes pp, public key pk and message $m \in \mathbb{G}$ as inputs. It samples a random integer $r \xleftarrow{\$} \mathbb{Z}_p^*$ and returns the ciphertext ct = $(\mathrm{ct}_1, \mathrm{ct}_2) = (\mathrm{G}^r, m \cdot \mathsf{pk}^r)$ as output.
- PKE.Dec(pp, sk, ct): It takes pp, the secret key sk and ciphertext ct as inputs. It then returns $m' = \mathrm{ct}_2/(\mathrm{ct}_1)^{\mathsf{sk}}$ as output.

The security of this construction relies on the hardness of DDH assumption over group $\mathbb{G}$. Over a bilinear group, if SXDH holds (DDH is hard in $\mathbb{G}_1$ and $\mathbb{G}_2$), like Type-III bilinear groups, then ElGamal encryption remains secure over source groups $(\mathbb{G}_1, \mathrm{G}_1, p)$ and $(\mathbb{G}_2, \mathrm{G}_2, p)$.

F.1.5 Pedersen Commitment. Commitment schemes enable a committer to commit to a hidden value by ensuring two main security properties: (perfectly) hiding and (computationally) binding. The hiding of the commitment ensures that no information about the hidden committed value is revealed and binding guarantees no committer can open the same commitment under two distinct messages. The Pedersen commitment [58] can be described as follows:

- COM.Setup(1^λ): Take security parameter, λ, as input. Sample $\mathrm{G} \xleftarrow{\$} \mathbb{G}$ and $\mathrm{H} \xleftarrow{\$} \mathbb{G}$. Return the public parameters pp = $(\mathbb{G}, p, \mathrm{G}, \mathrm{H})$ as output.
- COM.Com(pp, $m; \tau$): Take public parameters pp, a message $m \in \mathbb{Z}_\mathrm{p}$ and random opening τ as inputs. Output cm = $\mathrm{G}^m \mathrm{H}^\tau$.
- COM.Verify(pp, cm, m', τ'): Compute $\mathsf{cm}' = \mathrm{G}^{m'} \mathrm{H}^{\tau'}$. Return 1, if cm = cm′; otherwise return 0.

F.1.6 Generalized Pedersen Commitments. The Pedersen commitment can be extended to the Generalized Pedersen commitment that enables to commit to more than one message. To be more precise, the message space can be defined as $\mathcal{M} = \mathbb{Z}_p^n$, where n is an upper bound for the number of committed messages.

- COM.Setup($1^\lambda, n$): Take security parameter, λ and an integer n as inputs. Sample n+1 random generators $\mathrm{G}, \mathrm{H}_1, \mathrm{H}_2, \ldots, \mathrm{H}_n \xleftarrow{\$} \mathbb{G}^{(n+1)}$. Return the public parameters pp = $(\mathbb{G}, p, \mathrm{G}, \mathrm{H}_1, \ldots, \mathrm{H}_n)$ as output.
- COM.Com(pp, $\vec{m}, \tau$): Take the public parameters pp, a message vector $\vec{m} := (m_1, \ldots, m_n)$ and random opening $\tau \in \mathbb{Z}_p$ as inputs. Output cm = $\mathrm{G}^\tau \prod_{j=1} \mathrm{H}_i^{m_i}$.
- COM.Verify(pp, cm, $\vec{m}', \tau'$): Compute $\mathsf{cm}' = \mathrm{G}^{\tau'} \prod_{j=1}^n \mathrm{H}_i^{m_i'}$. Return 1 if cm = cm′ and 0 otherwise.

F.1.7 Inner-product predicate encryption by Okamoto-Takashima. We give a brief overview of what we briefly refer to as OT12 scheme [57]. While describing the full scheme is outside the scope of this overview section, we briefly describe the basics behind public-key generation, encryption and decryption. Assume we are in a bilinear group setting pp := $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, \mathrm{G}_1, \mathrm{G}_2)$ as before. We describe there the predicate-only version already adapted to the asymmetric pairing case that we use in our implementation.

Public key and master secret. We first sample an invertible matrix X of dimension $N = 4n$+2 (where n is the number of attributes) with elements in $\mathbb{F}_p^*$ and consider the matrix $\psi \cdot X^{-1}$ for a random, non-zero field element ψ. For syntactical purposes only, the transpose is actually considered, i.e., we define $Y = \psi \cdot (X^{-1})^T$.

For notational purposes, we define basis vectors $\vec{a}_i = (\underbrace{1_{\mathbb{G}_2}, \ldots, 1_{\mathbb{G}_2}}_{i-1}, \mathrm{G}_2, \underbrace{1_{\mathbb{G}_2}, \ldots, 1_{\mathbb{G}_2}}_{N-i})$ and $\vec{a}_i^* = (\underbrace{1_{\mathbb{G}_1}, \ldots, 1_{\mathbb{G}_1}}_{i-1}, \mathrm{G}_1, \underbrace{1_{\mathbb{G}_1}, \ldots, 1_{\mathbb{G}_1}}_{N-i})$. For a an element $c \in \mathbb{F}_p$ the notation $c\vec{a}_i$ is shorthand for $(1_{\mathbb{G}_2}, \ldots, 1_{\mathbb{G}_2}, \mathrm{G}_2^c, 1_{\mathbb{G}_2}, \ldots, 1_{\mathbb{G}_2})$. And for two vectors $\vec{b} = (B_1, \ldots, B_N) \in \mathbb{G}_i$ and $\vec{b}' = (B_1', \ldots, B_N') \in \mathbb{G}_i$, we write $\vec{b} \odot \vec{b}' := (B_1 B_1', \ldots, B_N B_N')$. Finally, a pairing operation continued for vectors is defined: let $\vec{c} = (C_1, \ldots, C_N) \in \mathbb{G}_1^N$ and $\vec{c}' = (C_1', \ldots, C_N') \in \mathbb{G}_2^N$, then $\hat{e}(\vec{c}, \vec{c}') := \prod_{i=1}^N e(C_i, C_i')$.

Armed with these tools and in particular the matrices $X = (x_{i,j})$ and $Y = (y_{i,j})$, we now define the public key and the master secret key: the public key $\mathbb{B} = (\vec{b}_1, \ldots, \vec{b}_N)$ consists of N vectors $\vec{b}_i :=$

$\bigodot_{j=1}^{N} x_{i,j}\vec{a}_j$. The master secret key $\mathbb{B}^* = (\vec{b}_1^*, \ldots, \vec{b}_N^*)$ consists of N vectors $\vec{b}_i^* := \bigodot_{j=1}^{N} y_{i,j}\vec{a}_j^*$.

We observe that there is the following relationship between $\mathbb{B}$ and $\mathbb{B}^*$ that follows from the definition of matrices X and Y:

$$\hat{e}(\vec{b}_i^*, \vec{b}_j) = e(\mathrm{G}_1, \mathrm{G}_2)^{x_{i,1}y_{j,1}+x_{i,2}y_{j,2}+\ldots+x_{i,N}y_{j,N}} = \begin{cases} e(\mathrm{G}_1, \mathrm{G}_2) =: \mathrm{G}_T, & \text{if } i = j, \\ 1_{\mathbb{G}_T}, & \text{if } i \neq j. \end{cases}$$

Key generation. For an attribute vector $\vec{v} \in \mathbb{F}_p^n \setminus \{\vec{0}\}$, one first samples $\sigma \leftarrow \mathbb{F}_p$ and $\vec{n} \leftarrow \mathbb{F}_p^n$ at random. We then form the vector $\vec{z}^* = (1, \sigma\vec{v}, \underbrace{0, \ldots, 0}_{2n}, \vec{n}, 0)$. The key for attribute $\vec{v}$ is defined as $\vec{k}^* := \bigodot_{i=1}^{N} z_i^*\vec{b}_i^*$.

Encryption. For encryption, which is done relative to attribute vector $\vec{x} \in \mathbb{F}_p^n \setminus \{\vec{0}\}$, one samples random values $\omega, \phi \leftarrow \mathbb{F}_p$ and defines the helper vector $\vec{z} := (1, \omega\vec{x}, \underbrace{0, \ldots, 0}_{3n}, \phi)$. The ciphertext is defined as $\vec{c} := \bigodot_{i=1}^{N} z_i\vec{b}_i$.

Decryption. In the predicate-only case, a key $\vec{k}^*$ decrypts a ciphertext $\vec{c}$ iff $\hat{e}(\vec{k}^*, \vec{c}) = \mathrm{G}_T$.

We observe, for correctness, that due to the above relation between $\mathbb{B}^*$ and $\mathbb{B}$, the operation $\hat{e}$ in fact computes the inner product of the vectors $\vec{z}^*$ and $\vec{z}$ in the exponent of G_T. That is, $\hat{e}(\vec{k}^*, \vec{c}) = e(\mathrm{G}_1, \mathrm{G}_2)^{1+\omega\sigma\langle\vec{v},\vec{x}\rangle}$, and therefore $\hat{e}(\vec{k}^*, \vec{c}) = \mathrm{G}_T$ when the inner product $\langle\vec{v}, \vec{x}\rangle$ is zero.

We refer to [57] for the proof that this scheme is attribute hiding in the sense defined in Appendix A.8.

F.2 Proof Systems

In this section, we give some background on the proof systems we use in our implementation with a selection of useful basic protocols.

We use the standard sigma protocols [60] as well as some recent techniques described in [30, 53]. To make the schemes non-interactive, we use the Fiat-Shamir paradigm [36] w.r.t. a hash functions $H : \{0,1\}^* \to \mathbb{Z}_p$. In essence, whenever the prover has the knowledge of scalar witnesses, we use sigma protocols to obtain an efficient zero-knowledge proof.

Additionally, we use Groth-Sahai (GS) proof systems [47] when the witness is a group element with hidden discrete logarithms. Over an asymmetric bilinear group $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, \mathrm{G}_1, \mathrm{G}_2)$, this construction can prove the satisfiability of any pairing-product equation (PPE) of the form $\prod_{i=1}^{n} e(A_i, \mathcal{Y}_i) \prod_{i=1}^{m} e(\mathcal{X}_i, B_i) \prod_{j=1}^{m} \prod_{i=1}^{n} e(\mathcal{X}_j, \mathcal{Y}_i)^{\gamma_{i,j}} = T$, where $\mathcal{X}_1, \ldots, \mathcal{X}_m \in \mathbb{G}_1$, $\mathcal{Y}_1, \ldots, \mathcal{Y}_n \in \mathbb{G}_2$ are the witnesses given as a commitment and $T \in \mathbb{G}_T$, $A_1, \ldots, A_n \in \mathbb{G}_1$, $B_1, \ldots, B_m \in \mathbb{G}_2$ and $\{\gamma_{i,j}\}_{i\in[1,m],j\in[1,n]} \in \mathbb{Z}_p$ are constant values which are a part of the instance or publicly known. Another advantage of using GS proofs is the ability to use verification batching techniques, such as the one described in this paper [49].

Finally, the range-proof [18] allows a user to prove that a hidden value lies within a certain range. In the proposed constructions, the number of re-randomizations is upper bounded by a maximum number T_{Rand} that is fixed in the setup. In order to prove that this condition is fulfilled, our instantiation relies on range-proofs proposed by Bünz et al. [19], known as Bulletproofs.

F.2.1 Sigma protocols. We first summarize the utilized sigma protocols (that is, the non-interactive versions via the Fiat-Shamir heuristic) in our efficient instantiation and give an overview on the techniques implied in their implementations. All the protocols are assumed a bilinear group setting $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, e, \mathrm{G}_1, \mathrm{G}_2)$ and the Pedersen commitment.

Σ-Dlog$\{(a) \mid A = \mathrm{G}^a\}$

- Prove(CRS, x, w): Takes the instance $x = (A)$ and the witness $w = (a)$ as inputs. It then samples $r \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $R = \mathrm{G}^r$ and challenge $c = H(A, R, \mathrm{G})$ and forms $z = r - ca \mod p$. It then returns the proof $\pi = (c, z, R)$ as output.
- Verify(CRS, x, π): Takes the instance $x = (A)$ and proof $\pi = (c, z, R)$ as inputs. It then computes $c' = H(A, R, \mathrm{G})$ and checks the equality of $c' = c$ and $R = A^c\mathrm{G}^z$. It returns 1 if they hold and 0 otherwise.

Figure 20: Non-interactive proof of knowledge of Dlog.

Proving the Knowledge of Discrete Logarithm. Figure 20 describes a non-interactive sigma protocol that enables a prover to prove the knowledge of a scalar witness $a \in \mathbb{Z}_p$ under the public instance of $A = \mathrm{G}^a$, where G is a generator of a group $\mathbb{G}$ of prime order p. Note that the hash function $H : \{0,1\}^* \to \mathbb{Z}_p$ is modeled in the random oracle model.

Σ-ElGamal$\{(m, r, e) \mid \mathsf{ct}_1 = \mathrm{G}_1^r \wedge \mathsf{ct}_2 = \mathrm{G}_1^m\mathsf{pk}^r \wedge \mathsf{cm} = \mathrm{G}^m\mathrm{H}^e\}$

- Prove(CRS, x, w): Takes the instance $x = (\mathsf{ct}_1, \mathsf{ct}_2, \mathsf{cm})$ and the witness $w = (m, r, e)$ as inputs. It then samples $r_1, r_2, r_3 \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $R_1 = \mathrm{G}_1^{r_1}$, $R_2 = \mathrm{G}_1^{r_2}\mathsf{pk}^{r_1}$ and $R_3 = \mathsf{COM.Com}(\mathsf{pp}, r_2; r_3) = \mathrm{G}^{r_2}\mathrm{H}^{r_3}$, the challenge $c = H(\mathsf{ct}_1, \mathsf{ct}_2, \mathsf{cm}, R_1, R_2, R_3, \mathrm{G}, \mathrm{H})$ and forms $z_1 = r_1 - cr \mod p$, $z_2 = r_2 - cm \mod p$ and $z_3 = r_3 - ce \mod p$. It then returns the proof $\pi = (c, z_1, z_2, z_3, R_1, R_2, R_3)$ as output.
- Verify(CRS, x, π): Takes the instance $x = (\mathsf{ct}_1, \mathsf{ct}_2, \mathsf{cm})$ and proof $\pi = (c, z_1, z_2, z_3, R_1, R_2, R_3)$ as inputs. It then computes $c' = H(\mathsf{ct}_1, \mathsf{ct}_2, \mathsf{cm}, R_1, R_2, R_3, \mathrm{G}, \mathrm{H})$ and checks the equality of equations $c' = c$, $R_1 = \mathsf{ct}_1^c\mathrm{G}_1^{z_1}$, $R_2 = \mathrm{G}_1^{z_2}\mathsf{pk}^{z_1}\mathsf{ct}_2^c$ and $R_3 = \mathrm{G}^{z_2}\mathrm{H}^{z_3}\mathsf{cm}^c$. It returns 1 if all the equations hold; 0 otherwise.

Figure 21: Non-interactive proof of knowledge of ElGamal encrypted value.

Proving the Knowledge of a Committed value and its ElGamal encryption. Figure 21 describes a sigma protocol proving the knowledge of a palintext m encrypted based on ElGamal encryption and committed via Pedersen commitment. In other words, proving the

knowledge of scalar message $m \in \mathbb{Z}_p$ such that $\mathsf{cm} = \mathsf{G}^m\mathsf{H}^e$ and simultanously we have, $\mathsf{ct} = (\mathsf{ct}_1, \mathsf{ct}_2) = (\mathsf{G}_1^r, \mathsf{G}_1^m\mathsf{pk}^r)$. The hash function $H : \{0,1\}^* \to \mathbb{Z}_p$ is modeled in the random oracle model.

Proving the Equality of Committed Values in different groups. We extend the protocol proposed in [30] s.t. for a given cyclic groups $\mathbb{G}_1$ and $\mathbb{G}_2$ of prime order p, Figure 22 describes a sigma protocol enabling a prover to prove that two commitments $\mathsf{cm}_1 = \mathsf{G}_1^m\mathsf{H}_1^{e_1}\mathsf{K}_1^{u_1}$ and $\mathsf{cm}_2 = \mathsf{G}_2^m\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}$ are committing to the same message m. Note that $\mathsf{G}_1, \mathsf{H}_1, \mathsf{K}_1 \in \mathbb{G}_1$ and $\mathsf{G}_2, \mathsf{H}_2, \mathsf{K}_2 \in \mathbb{G}_2$ s.t. the discrete logarithms $\log_{\mathsf{G}_1}(\mathsf{H}_1)$, $\log_{\mathsf{G}_1}(\mathsf{K}_1)$ and $\log_{\mathsf{G}_2}(\mathsf{H}_2)$, $\log_{\mathsf{G}_2}(\mathsf{K}_2)$ are unknown to the prover. The hash function $H' : \{0,1\}^* \to \mathbb{Z}_{2^k}$, where k is a fixed integer and $2^k < p$ is modeled in the random oracle model.

$\Sigma\text{-Bridging}\{(m, e_1, e_2, u_1, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^m\mathsf{H}_1^{e_1}\mathsf{K}_1^{u_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^m\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$

•$\mathsf{Prove}(\mathsf{CRS}, x, w)$: Takes the instance $x = (\mathsf{cm}_1, \mathsf{cm}_2)$ and the witness $w = (m, e_1, e_2, u_1, u_2)$ as inputs. It then samples $r_1, r_2, r_3, r_4, r_5 \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $R_1 = \mathsf{G}_1^{r_1}\mathsf{H}_1^{r_2}\mathsf{K}_1^{r_3}$, $R_2 = \mathsf{G}_2^{r_1}\mathsf{H}_2^{r_4}\mathsf{K}_2^{r_5}$ and the challenge $c = H'(\mathsf{cm}_1, \mathsf{cm}_2, R_1, R_2, \mathsf{G}_1, \mathsf{H}_1, \mathsf{K}_1, \mathsf{G}_2, \mathsf{H}_2, \mathsf{K}_2)$ and forms $z_1 = r_1 - cm \mod p$, $z_2 = r_2 - ce_1 \mod p$, $z_3 = r_3 - cu_1 \mod p$, $z_4 = r_4 - ce_2 \mod p$ and $z_5 = r_5 - cu_2 \mod p$. It then returns the proof $\pi = (c, z_1, z_2, z_3, z_4, z_5, R_1, R_2)$ as output.

•$\mathsf{Verify}(\mathsf{CRS}, x, \pi)$: Takes the instance $x = (\mathsf{cm}_1, \mathsf{cm}_2)$ and proof $\pi = (c, z_1, z_2, z_3, z_4, z_5, R_1, R_2)$ as inputs. It then computes $c' = H'(\mathsf{cm}_1, \mathsf{cm}_2, R_1, R_2, \mathsf{G}_1, \mathsf{H}_1, \mathsf{K}_1, \mathsf{G}_2, \mathsf{H}_2, \mathsf{K}_2)$ and checks the equality of $c' = c$, $R_1 = \mathsf{cm}_1^c\mathsf{G}_1^{z_1}\mathsf{H}_1^{z_2}\mathsf{K}_1^{z_3}$ and $R_2 = \mathsf{cm}_2^c\mathsf{G}_2^{z_1}\mathsf{H}_2^{z_4}\mathsf{K}_2^{z_5}$. It returns 1 if the checks hold and 0 otherwise.

Figure 22: Non-Interactive proof of Equality of Two Commitments.

Proving a Multiplicative Relation on Committed Values. Figure 23 describes a sigma protocol to prove the knowledge of committed values and also a multiplicative relation between them. More precisely, this protocol enables to prove the knowledge of integers x_1 and x_2 s.t. $x_3 = x_1x_2 \mod p$ by issuing the commitments $\mathsf{cm}_i = \mathsf{G}^{x_i}\mathsf{H}^{e_i}$ for $i = 1, 2, 3$. Note that the hash function $H' : \{0,1\}^* \to \mathbb{Z}_{2^k}$, where k is a fixed integer and $2^k < p$ is modeled in the random oracle model.

$\Sigma\text{-MultCom}\{\{(x_j, e_j)\}_{j=1}^3 \mid \mathsf{cm}_i = \mathsf{G}^{x_i}\mathsf{H}^{e_i} \text{ for } i = 1,2,3 \wedge x_3 = x_1x_2 \mod p\}$

•$\mathsf{Prove}(\mathsf{CRS}, x, w)$: Takes the instance $x = (\mathsf{cm}_1, \mathsf{cm}_2, \mathsf{cm}_3)$ and the witness $w = (x_1, x_2, x_3, e_1, e_2, e_3)$ as inputs. It then samples $r_1, r_2, r_3 \xleftarrow{\$} \mathbb{Z}_p^*$ and $s, s_1, s_2, s_3 \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $R_i = \mathsf{G}^{r_i}\mathsf{H}^{s_i}$ and $R = \mathsf{cm}_1^{r_2}\mathsf{H}^s$, the challenge $c = H'(\mathsf{cm}_1, \mathsf{cm}_2, \mathsf{cm}_3, R, R_1, R_2, R_3, \mathsf{G}, \mathsf{H})$ and forms $z_i = r_i - cx_i \mod p$, $t_i = s_i - ce_i \mod p$ for $i = 1, 2, 3$ and $t = s - ce \mod p$, where $e = e_3 - e_1x_2 \mod p$. It then returns the proof $\pi = (c, z_1, z_1, z_3, t_1, t_2, t_3, t)$ as output.

•$\mathsf{Verify}(\mathsf{CRS}, x, \pi)$: Takes the instance $x = (\mathsf{cm}_1, \mathsf{cm}_2, \mathsf{cm}_3)$ and proof $\pi = (c, z_1, z_1, z_3, t_1, t_2, t_3, t)$ as inputs. It then computes $c' = H'(\mathsf{cm}_1, \mathsf{cm}_2, \mathsf{cm}_3, R, R_1, R_2, R_3, \mathsf{G}, \mathsf{H})$ and checks the equality of equations $c' = c$, $R_i = \mathsf{G}^{z_i}\mathsf{H}^{t_i}\mathsf{cm}_i^c$ for $i = 1, 2, 3$ and $R = \mathsf{cm}_3^c\mathsf{cm}_1^{z_2}\mathsf{H}^t$. It returns 1 if they hold and 0 otherwise.

Figure 23: Non-Interactive proof of multiplicative relation on committed values.

Proving the Knowledge of a DY PRF key and its Well-Formedness. Figure 24 recalls the DY PRF well-formedness protocol described in [30]. As part of this protocol, a prover using DY PRF key k shows that the PRF output is formed correctly under a given input ctr, i.e. $\mathsf{ID} = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr}) = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}$.

$\Sigma\text{-PRF}\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID} = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\}$

•$\mathsf{Prove}(\mathsf{CRS}, x, w)$: Takes the instance $x = (\mathsf{ID})$ and the witness $w = (\mathsf{k}, \mathsf{ctr})$ as inputs. Note that ID can be seen as a commitment of the form $\mathsf{COM.Com}(\mathsf{pp}, (1/\mathsf{k}+\mathsf{ctr}); 0) = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\mathsf{H}^0$. It samples $e_1, e_2, e_3 \xleftarrow{\$} \mathbb{Z}_p^*$ and computes the commitments $\mathsf{cm}_1 = \mathsf{COM.Com}(\mathsf{pp}, \mathsf{ctr}; e_1) = \mathsf{G}^{\mathsf{ctr}}\mathsf{H}^{e_1}$, $\mathsf{cm}_2 = \mathsf{COM.Com}(\mathsf{pp}, \mathsf{k}; e_2) = \mathsf{G}^{\mathsf{k}}\mathsf{H}^{e_2}$, $\mathsf{cm}_3 = \mathsf{COM.Com}(\mathsf{pp}, (\mathsf{k}+\mathsf{ctr}); e_3) = \mathsf{G}^{(\mathsf{k}+\mathsf{ctr})}\mathsf{H}^{e_3}$. It then runs $\pi \leftarrow \Sigma\text{-MultCom.Prove}(\mathsf{CRS}, x_1, w_1)$ with input commitments $x_1 = (\mathsf{cm}_1 \cdot \mathsf{cm}_2, \mathsf{ID}, \mathsf{G})$ and $w_1 = (\mathsf{k}+\mathsf{ctr}, 1/(\mathsf{k}+\mathsf{ctr}), 1)$. It returns the proof π as output.

•$\mathsf{Verify}(\mathsf{CRS}, x, \pi)$: Takes the instance $x = (\mathsf{cm}_1 \cdot \mathsf{cm}_2, \mathsf{ID}, \mathsf{G})$ and proof π as inputs. It then checks the validity of the proof by running $\Sigma\text{-MultCom.Verify}(\mathsf{CRS}, x, \pi)$.

Figure 24: Non-interactive proof of knowledge of DY's PRF key and its well-formedness.

Proving the Knowledge of opening of Generalized Pedersen Commitment. Figure 25 recalls the proving knowledge of opening in a Generalized Pedersen commitment protocol described in [46]. As part of this protocol, a prover using a vector of messages $\vec{m}$ shows that the commitment cm is computated correctly and it has the

knowledge of opening τ under a given public parameter pp, i.e. $\mathsf{cm} = \mathsf{COM.Com}(\mathsf{pp}, \vec{m}, \tau) = \mathsf{G}^{\tau} \prod_{i=1}^{n} \mathsf{H}_i^{m_i}$.

Σ-GPedCom$\{(m_1, \ldots, m_n, \tau) \mid \mathsf{cm} = \mathsf{G}^{\tau} \prod_{i=1}^{n} \mathsf{H}_i^{m_i}\}$

•Prove(CRS, x, w): Takes the instance $x = (\mathsf{cm}, \mathsf{G}, \mathsf{H}_1, \ldots, \mathsf{H}_n)$ and the witness $w = (m_1, \ldots, m_n, \tau)$ as inputs. It samples $\vec{x} = (x_1, \ldots, x_n) \xleftarrow{\$} \mathbb{Z}_p^n$ and $\tau_x \leftarrow \mathbb{Z}_p$ and computes $\mathsf{cm}_x = \mathsf{G}^{\tau_x} \prod_{i=1}^{n} \mathsf{H}_i^{x_i}$. It then computes the challenge $c = H'(\mathsf{cm}, \mathsf{cm}_x, \mathsf{G}, \mathsf{H}_1, \ldots, \mathsf{H}_n)$ and forms $z_i = x_i + c \cdot m_i \mod p$ for $i \in [1, n]$ and $t = \tau_x + c \cdot \tau \mod p$. It returns the proof $\pi = (\mathsf{cm}_0, z_1, \ldots, z_n, t)$ as output.

•Verify(CRS, x, π): Takes the instance $x = (\mathsf{cm}, \mathsf{G}, \mathsf{H}_1, \ldots, \mathsf{H}_n)$ and proof $\pi = (\mathsf{cm}_0, z_1, \ldots, z_n, t)$ as inputs. It then computes $c' = H'(\mathsf{cm}, \mathsf{cm}_x, \mathsf{G}, \mathsf{H}_1, \ldots, \mathsf{H}_n)$ and checks the equality of equations $c' = c$, $\mathsf{cm}_x \cdot \mathsf{cm}^c = \mathsf{COM.Com}(\vec{z}, t) = \mathsf{G}^t \prod_{i=1}^{n} \mathsf{H}_i^{z_i}$. It returns 1 if they hold and 0 otherwise.

Figure 25: Non-Interactive proof of knowledge of opening of Generalized Pedersen commitments.

F.2.2 Groth-Sahai proofs. GS proofs [47] are able to prove the satisfiability of some quadratic equations in bilinear setting. However, in this paper, we only use GS proofs to demonstrate pairing product equations satisfiability of the following form,

$$\prod_{i=1}^{n} e(A_i, \mathcal{Y}_i) \prod_{i=1}^{m} e(\mathcal{X}_i, B_i) \prod_{j=1}^{m} \prod_{i=1}^{n} e(\mathcal{X}_j, \mathcal{Y}_i)^{\gamma_{i,j}} = T \ ,$$

where $\mathcal{X}_1, \ldots, \mathcal{X}_m \in \mathbb{G}_1, \mathcal{Y}_1, \ldots, \mathcal{Y}_n \in \mathbb{G}_2$ are the witnesses given as a commitment and $T \in \mathbb{G}_T$, $A_1, \ldots, A_n \in \mathbb{G}_1$, $B_1, \ldots, B_m \in \mathbb{G}_2$ and $\Gamma := \{\gamma_{i,j}\}_{i \in [1,m], j \in [1,n]} \in \mathbb{Z}_p^{m \times n}$.

GS proofs are essentially commit-and-prove systems, in which the prover proves that a quadratic equation satisfies using the committed assignments. Therefore, there are two steps: first, the prover commits to the values, and then it proves their validity through some relation. This scheme can be instantiate in two possible settings: non-interactive witness-indistinguishable (NIWI) and non-interactive zero-knowledge (NIZK). If in the described PPE, the constant value $T = 1_{\mathbb{G}_T}$ this construction can guarantee Zero-Knowledge property[8]. Thus in the rest of this section we take this condition into account. In the following, we briefly summarize the most efficient instantiation of GS proofs based on SXDH assumption (i.e. DDH holds in both source groups $\mathbb{G}_1$ and $\mathbb{G}_2$), both in terms of proof size and number of basic pairings required in the verification phase.[9]

Extended bilinear maps, $E : \mathbb{G}_1^2 \times \mathbb{G}_2^2 \to \mathbb{G}_T^4$, are a generalization for the standard bilinear pairings, defined in Definition A.1. For any given group elements $a_1, a_2 \in \mathbb{G}_1$ and $b_1, b_2 \in \mathbb{G}_2$, an extended bilinear map (tensor product) is defined as follows:

$$E\left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} b_1 & b_2 \end{pmatrix}\right) = \begin{pmatrix} e(a_1, b_1) & e(a_1, b_2) \\ e(a_2, b_1) & e(a_2, b_2) \end{pmatrix} \ .$$

GS proofs additionally rely on a variation of the Pedersen commitments, which are discussed in Appendix F.1, where the commitments are generated based on two generators rather than a single one. Loosely speacking, this special commitment scheme enables proof simulation. The double generator Pedersen commitment over a cyclic group $\mathbb{G} = \langle \mathsf{G} \rangle$ with a prime order p consists of the following PPT algorithms:

- $\mathsf{pp} \leftarrow \mathsf{COM.Setup}(1^{\lambda})$: Take security parameter, λ in its unary representation as input. Sample $\mathsf{H}_1, \mathsf{H}_2 \xleftarrow{\$} \mathbb{G}$. Return the public parameters $\mathsf{pp} = (\mathbb{G}, p, \mathsf{G}, \mathsf{H}_1, \mathsf{H}_2)$ as output.
- $\mathsf{cm} \leftarrow \mathsf{COM.Com}(\mathsf{pp}, M; \tau_1, \tau_2)$: Take public parameters pp, a message $M \in \mathbb{G}$ and random openings τ_1, τ_2 as inputs. Output $\mathsf{cm} = M\mathsf{H}_1^{\tau_1}\mathsf{H}_2^{\tau_2}$.
- $0/1 \leftarrow \mathsf{COM.Verify}(\mathsf{pp}, \mathsf{cm}, M', \tau_1', \tau_2')$: Compute $\mathsf{cm}' = M'\mathsf{H}_1^{\tau_1'}\mathsf{H}_2^{\tau_2'}$. Return 1, if $\mathsf{cm} = \mathsf{cm}'$; otherwise return 0.

Next we outline GS proofs for a simple case based on SXDH assumption that the prover aims to prove the knowledge of group elements $\mathcal{X}, \mathcal{Y} \in \mathbb{G}_1 \times \mathbb{G}_2$ as witnesses s.t. $e(\mathcal{X}, \mathcal{Y})^{\gamma} = T$, where $T \in \mathbb{G}_T$ is a known constant value.

- $\mathsf{CRS} \leftarrow \mathsf{GS.Setup}(1^{\lambda})$: Take the security parameter λ as input. Sample eight group elements $\mathsf{CRS} := \mathsf{H}_1, \mathsf{H}_2, \mathsf{K}_1, \mathsf{K}_2, \mathsf{U}_1, \mathsf{U}_2, \mathsf{V}_1, \mathsf{V}_2) \xleftarrow{\$} \mathbb{G}_1^4 \times \mathbb{G}_2^4$. It then returns CRS as output.
- $\pi \leftarrow \mathsf{GS.Prove}(\mathsf{CRS}, x, w)$. It takes CRS, witness $w = (\mathcal{X}, \mathcal{Y}) \in \mathbb{G}_1 \times \mathbb{G}_2$ and instance $x = (T)$ as inputs. It samples the random integers $r_1, r_2, s_1, s_2 \xleftarrow{\$} \mathbb{Z}_p^*$ and commits to witness by computing $(a_1, a_2) = \left(\mathsf{H}_1^{r_1}\mathsf{H}_2^{r_2}, \mathcal{X}\mathsf{K}_1^{r_1}\mathsf{K}_2^{r_2}\right)$ and $(b_1, b_2) = \left(\mathsf{U}_1^{s_1}\mathsf{U}_2^{s_2}, \mathcal{Y}\mathsf{V}_1^{s_1}\mathsf{V}_2^{s_2}\right)$. It samples the random integers $\alpha, \beta, \zeta, \delta \xleftarrow{\$} \mathbb{Z}_p$ and then generates the proofs $\phi_1 = (b_1^{\gamma r_1}\mathsf{U}_1^{\alpha}\mathsf{V}_1^{\beta} \quad b_2^{\gamma r_2}\mathsf{U}_2^{\alpha}\mathsf{V}_2^{\beta})$, $\phi_2 = (b_1^{\gamma s_1}\mathsf{U}_1^{\zeta}\mathsf{V}_1^{\delta} \quad b_2^{\gamma s_2}\mathsf{U}_2^{\zeta}\mathsf{V}_2^{\delta})$, $\theta_1 = (\mathsf{H}_1^{-\alpha}\mathsf{K}_1^{-\zeta} \quad \mathcal{X}^{r_2}\mathsf{H}_2^{-\alpha}\mathsf{K}_2^{-\zeta})$ and $\theta_2 = (\mathsf{H}_1^{-\beta}\mathsf{K}_1^{-\delta} \quad \mathcal{X}^{\gamma s_2}\mathsf{H}_2^{-\beta}\mathsf{K}_2^{-\delta})$, and return the proof $\pi = (a_1, a_2, b_1, b_2, \phi_1, \phi_2, \theta_1, \theta_2)$ as output.
- $0/1 \leftarrow \mathsf{GS.Verify}(\mathsf{CRS}, x, \pi)$: It takes CRS, the instance x and proof π as inputs. It then checks the validity of the following pairing product equation:

$$E\left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} b_1 & b_2 \end{pmatrix}^{\gamma}\right) = E\left(\begin{pmatrix} \mathsf{H}_1 \\ \mathsf{H}_2 \end{pmatrix}, \quad \phi_1\right) E\left(\begin{pmatrix} \mathsf{K}_1 \\ \mathsf{K}_2 \end{pmatrix}, \quad \phi_2\right) E\left(\theta_1, \quad \begin{pmatrix} \mathsf{U}_1 & \mathsf{U}_2 \end{pmatrix}\right) E\left(\theta_2, \quad \begin{pmatrix} \mathsf{V}_1 & \mathsf{V}_2 \end{pmatrix}\right) \begin{pmatrix} T & 1_{\mathbb{G}_T} \\ 1_{\mathbb{G}_T} & 1_{\mathbb{G}_T} \end{pmatrix} \ .$$

If the equation holds it returns 1 and accepts the proof; 0 otherwise.

[8] According to Escala and Groth [34], the proof system remains zero-knowledge if the base element for the group and public constant are paired to each other.

[9] To implement the Groth-Sahai proofs, we modified the GS implementation provided in this repository.

This simple example results in a single $\gamma \in \{-1, 0, 1\}$ value because $m = n = 1$. In general, however, Γ is a matrix of dimension $m \times n$. Throughout the next section, we discuss how this matrix can be defined for different PPEs.

Herold et al.'s batching technique. To check the validity of a GS proof for any PPE consisting of n first-group elements and m second-group elements, a verifier must compute $4(n + m + 4)$ pairings. However, Herold et al. described a batching technique in [49] that reduces the number of pairings by a factor of 4. This means that a verifier checking the same proof needs to compute only $n + m + 4$ pairings, which is a significant improvement, especially in real-world use cases. The authors replace an extended pairing product equation to a basic pairing product equation using linear algebra. As a simple example, for a given group vectors $\vec{a} \in \mathbb{G}_1^2$ and $\vec{b} \in \mathbb{G}_2^2$ the extended bilinear equation can be written as follows:

$$E\left(\vec{a}, \quad \vec{b}\right) = \begin{pmatrix} e(a_1, b_1) & e(a_1, b_2) \\ e(a_2, b_1) & e(a_2, b_2) \end{pmatrix} = \begin{pmatrix} t_1 & t_2 \\ t_3 & t_4 \end{pmatrix} \ ,$$

where $t_i \in \mathbb{G}_T$ for $i \in [1, 4]$. A verifier computes $A = a_1^{r_1} a_2^{r_2}$ and $B = b_1^{s_2} b_2^{s_2}$ using the randomnesses $r_1, r_2, s_1, s_2 \in \mathbb{Z}_p^*$. In this case, the above extended pairing product equation can be rewritten as follows:

$$e(A, B) = t_1^{r_1 s_1} t_2^{r_1 s_2} t_3^{r_2 s_1} t_4^{r_2 s_2} \ .$$

It is easy to see that the above technique is correct. However, the soundness error is at most $2/p$. More interestingly, it reduces the number of pairings by 75% compared to the naive approach. Specifically, it only requires a single pairing instead of 4 pairings.

F.2.3 Range-proofs. Range-proofs enable a prover to prove a committed value x computed as $\mathsf{cm} = \mathsf{COM.Com}(\mathsf{pp}, x, r)$ is in the range of $[0, 2^n)$.[10]

G REALIZATION OF NIZK RELATIONS

Next, we give a detailed description of the languages in the proposed ul-PCS constructions and the used techniques for their implementation. Note that for the ease of following we use the *gray* background to highlight the hidden values that should be considered as witnesses in each relation. Additionally, the described relations are given solely on their own, while the prover is expected to make a bridge between them. As the underlying proof systems rely on the commit-and-prove principle, and a commitment to a witness is issued by the prover, we can bridge the relations by applying the sigma protocol described in Figure 22 whenever a hidden parameter is used in more than one relation. In what follows, a bridging proof is indicated by ⧫. As it is illustrated in Figure 6, the realtions are proved with three main proof systems, including sigma protocols, range-proofs and Groth-Sahai (GS) proofs

G.1 Generic ul-PCS instantiated with Inner-Product Predicate Encryption

G.1.1 Language $\mathcal{L}_1$. The first language in the generic ul-PCS takes the instances $x_{\mathsf{st}} = (T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ct}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{A}, \mathsf{mpk}_{\mathsf{PE}})$ and the witness $w_{\mathsf{st}} := (\mathsf{k}, \mathsf{ctr}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, x, \sigma_{\mathsf{sig}}^1, \sigma_{\mathsf{sig}}^2, \sigma_{\mathsf{ctr}})$ as inputs and the prover proves the satisfiability of the following relations:

$\mathcal{L}_1$**.1**. $\boxed{\mathsf{ID}_{\mathsf{ctr}} = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})}$: As discussed on Appendix F.1, we use the DY PRF schemes. Thus we use the sigma protocol described in Figure 24 to prove its well-formedness, i.e. $\Sigma\text{-PRF}\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_{\mathsf{ctr}} = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\}$.

$\mathcal{L}_1$**.2**. $\mathsf{ctr} < T_{\mathsf{Rand}}$: Additionally, the prover utilizes the range-proof techniques to prove $\mathsf{ctr} \in [0, T_{\mathsf{Rand}})$.

⧫ To show that the used ctr in the above proofs are the same, the prover use the bridging sigma protocols described in Figure 22. More precisely, it runs $\Sigma\text{-Bridging}\{(\mathsf{ctr}, e_1, e_2, 0, 0) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{ctr}} \mathsf{H}_1^{e_1} \ \wedge \ \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{ctr}} \mathsf{H}_2^{e_2}\}$, where cm_1 is obtained via Σ-PRF protocol while cm_2 is computed by the range-proof protocol. Note that the generators $\mathsf{G}_1, \mathsf{H}_1, \mathsf{G}_2, \mathsf{H}_2$ are random elements of any cyclic group.

$\mathcal{L}_1$**.3**. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{k}, x), \sigma_{\mathsf{sig}}^1) = 1$: We instantiate this signature with the recalled SPS scheme in Appendix F.1. To prove the knowledge of a valid SPS signature σ_{sig}^1 on hidden message $\vec{M} = (\mathsf{G}_1^{\mathsf{k}}, \mathsf{G}_1^{x})$ that is signed by the CA can be written as a PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right) e\left(\mathsf{G}_1^{x}, \hat{X}_2^A\right) = e\left(R_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1\right) \ \wedge \ e(S_{\mathsf{sig}}^1, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^1)$, where $\mathsf{vk}_{\mathsf{sig}}^A := (\hat{X}_1^A, \hat{X}_2^A)$ and $\sigma_{\mathsf{sig}}^1 := (R_{\mathsf{sig}}^1, S_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1)$. We use GS proof systems to show the satisfiability of this equation.

⧫ To demonstrate that the used k in the first relation and the obove relation are the same, the prover use the bridging sigma protocols described in Figure 22. More precisely, it runs $\Sigma\text{-Bridging}\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}} \mathsf{H}_1^{e_1} \ \wedge \ \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}} \mathsf{H}_2^{e_2} \mathsf{K}_2^{u_2}\}$, where cm_1 is obtained via Σ-PRF protocol while cm_2 is computed by the GS proof systems.

$\mathcal{L}_1$**.4**. $\boxed{\mathsf{ct}_{\mathsf{ctr}} = \mathsf{PE.Enc}(\mathsf{mpk}_{\mathsf{PE}}, x)}$: To prove the well-formedness of the ciphertext $\mathsf{ct}_{\mathsf{ctr}}$ obtained from $\mathsf{PE.Enc}(\mathsf{mpk}_{\mathsf{PE}}, x)$ algorithm in the key re-randomization phase and demonstrate the fact that the attributes x are certified by the CA and folded with the PRF seed k, i.e. $\mathsf{SPS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{G}_1^{\mathsf{k}}, \mathsf{G}_1^{x}), \sigma_{\mathsf{sig}}^1) = 1$, we must make a few observations. Recall that the statements together should assure that the ciphertext is a correct encryption of some attribute x, and that this attribute is linked to the particular party's actual seed k (which it uses to provable derive its public pseudo-random identifier). The trick to obtain an implementation of this is fourfold:

(a) We employ OT12 as our POPE scheme described in Appendix F.1 and observe that the computation of the ciphertext $\vec{c} := \bigodot_{i=1}^{N} z_i \vec{b}_i$, which means in a component-wise notation that $c_i = \vec{b}_1[i]_1^z \cdot \vec{b}_2[i] \cdot \cdots \cdot \vec{b}_N[i]^{z_N}$, for $\vec{z} := (1, x_1, \ldots, x_n, \underbrace{0, \ldots, 0}_{3n}, \phi)$. This computation is very close to a generalized Pedersen commitment w.r.t. the vector of generators $(\vec{b}_1[i], \ldots, \vec{b}_N[i])$.

(b) Generalized Pedersen Commitments have a homomorphic property, and thus it is easy to, for a commitment cm

[10] To implement the range-proof, we use the open-source bulletproof Python implementation available in this repository.

to some vector $\vec{x}$, a commitment to $\omega\vec{x}$ by raising the commitment to the power of ω.

(c) The next step is to connect it to SPS-EQ in order to transfer the issuance of attributes by the authority to re-randomizations following the idea used in the role-based scheme (cf. Appendix D.2 and cf. also Appendix G.3). This means that if the authority issues an initial OT12 ciphertext, in the form of N generalized Pedersen commitments $\mathsf{cm}_{\vec{x},i}$ that are blindings of the c_i values above for the vector $\vec{z} := (1, x_1, \ldots, x_n, \underbrace{0, \ldots, 0}_{3n}, 0)$, together with an SPS-EQ signature on that vector, then a party is able to generate, using the homomorphic property, a commitment to a scaled vector on its attributes $\mathsf{cm}_{\omega\vec{x},i}$ (as required by OT12), and by the signature adaptation and unforgeability property of the SPS-EQ scheme it can indeed be verified that this is done correctly. By using the same trick as in the role-based scheme, we can further bind this vector specifically to a party (see next point).
To randomize the ciphertext correctly according to OT12, we further compute a commitment to a vector $\vec{\phi}_i = (\underbrace{1, 0, \ldots, 0}_{n}, \phi_i, \underbrace{0, \ldots, 0}_{3n-1}, \phi')$, where $\phi_i, \phi_2 \xleftarrow{\$} \mathbb{Z}_p^*$ and $n = [\frac{N-2}{4}]$, and prove knowledge of the opening (cf. Figure 25), in particular, this includes that we verify that the zero-positions are indeed zero (or alternatively, that is indeed a vector of length 3) [46]. The Generalized Pedersen commitments of this vector under the same basis denoted by $\vec{\mathsf{cm}}_\phi = \left(\mathsf{cm}_{\phi_1}, \ldots, \mathsf{cm}_{\phi_N}\right)$ and can be homomorphically combined with the N commitments $\mathsf{cm}_{\omega\vec{x},i}$ to yield N commitments cm_i^{OT12} to the OT12 ciphertext components ct_i, where each component is now encoding the vector $\vec{z}_i := (1, x_1, \ldots, x_n, \phi_i, \underbrace{0, \ldots, 0}_{3n-1}, \phi')$. We reveal ct_i by revealing the final randomness of the Pedersen commitment. We further need to prove knowledge of the committed vector (using Figure 25 where the verifier can use the revealed r_i directly) in order to be formally extractable.[11] Note that some care must be taken when revealing the r_i, which is why we enocde vectors $\vec{z}_i := (1, x_1, \ldots, x_n, \phi_i, \underbrace{0, \ldots, 0}_{3n-1}, \phi')$ and not just $(1, x_1, \ldots, x_n, \underbrace{0, \ldots, 0}_{3n}, \phi')$. The additional randomness contribution, ϕ_i injected at a position that does not affect the inner-product computation of OT12 ensures that we can reveal r_i (masking the ith component) without leaking information about the intermediate computations relevant for OT12 in the commitments cm_{ϕ_i} and $\mathsf{cm}_{\omega\vec{x},i}$, in particular, this can be thought of as masking ϕ'.

[11] This step could be omitted in an implementation to improve efficiency while trading provable for heuristic security. This appears acceptable in environments where a CRS is established using a ceremony to ensure that no trapdoor does exist in the system.

Now, all ingredients are in place: a verifier is able to retrace the computation (where we put all elements required to do so in the proof string), and verify the SPS-EQ signature to be sure the OT12 ciphertext is correctly formed and connected to the attribute that was issued to the party.

(d) Finally, in order to link the party's seed k to this vector, we apply the same trick as in the role-based scheme (cf. Appendix D.2 and Appendix G.3) and create an accumulator $\mathsf{A_k}$ to which we add k (cf. Appendix A.6). We add the pair $(\mathsf{A_k}, \mathsf{G}_2, \ldots)$ to the above vector. As shown in Appendix D.2 and Appendix G.3, re-randomizing both elements $(\mathsf{A_k}, \mathsf{G}_2)$ preserves the relationship to prove that an element, in this case k, is in the accumulator. Thus, the party cannot only present a randomized vector (which is a commitment to the scaled attributes $\vec{x}$ as required by OT12), but also that this vector has been issued in connection with the seed k (which it uses to develop the PRF as described below) by proving that it has the corresponding accumulator witness. This completes the high-level realization of the two assertions above.
♦ To demonstrate that the used $\mathsf{A_k}$ in the vector $(\mathsf{A_k}, \ldots)$ is a valid accumulator value under the same PRF seed k in the first relation we first run a GS proof to prove the accumulator verification holds under k. Additionally we use the bridging sigma protocols described in Figure 22 to show this seed is the same as the one in the first relation on the well-formedness of PRF. More precisely, it runs $\Sigma\text{-Bridging}\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$, where cm_1 is obtained via Σ-PRF protocol while cm_2 is computed by the GS proof systems on the validity of the accumulator verification.

$\mathcal{L}_1$**.5**. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}), \sigma_{\mathsf{sig}}^2) = 1$: As we discuss in Appendix F.1, this signature scheme is instantiated by a SPS. The knowledge of a SPS signature σ_{sig}^2 on hidden message $\vec{M} = (\mathsf{G}_1^{\mathsf{k}}, \mathsf{vk}_{\mathsf{sig}})$ that is signed by the CA can be written as a PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right) e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^A\right) = e\left(R_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2\right) \wedge e(S_{\mathsf{sig}}^2, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^2)$, where $\mathsf{vk}_{\mathsf{sig}}^A := (\hat{X}_1^A, \hat{X}_2^A)$ and $\sigma_{\mathsf{sig}}^2 := (R_{\mathsf{sig}}^2, S_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2)$. We use GS proof systems to show the satisfiability of this equation.

♦ The prover additionally runs $\Sigma\text{-Bridging}\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$, where cm_1 is obtained via Σ-PRF protocol while cm_2 is computed by the GS proof systems on the knowledge of SPS signature σ_{sig}^2.

$\mathcal{L}_1$**.6**. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}, (\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ID}_{\mathsf{ctr}}), \sigma_{\mathsf{ctr}}) = 1$: This signature is instantiated by the BLS signature, discussed in Appendix F.1. The prover should prove the satisfiability of the PPE relation described below in order to validate a newly generated verification key, $\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}$, and to bind it with the new identifier $\mathsf{ID}_{\mathsf{ctr}}$, $e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}} || \mathsf{ID}_{\mathsf{ctr}})\right) = e(\mathsf{G}_1, \sigma_{\mathsf{ctr}})$ that represents the validity of BLS signature. We use GS proof systems to instantiate this relation in zero-knowledge.

♦ To show that the $\mathsf{vk}_{\mathsf{sig}}$ element in the relations discussed in $\mathcal{L}_1.5$ and $\mathcal{L}_1.6$ are identical we need to make a bridge between them. Due to the fact that both of these relations are proven via GS proof systems, we can instead combine them as follows [47]:

$$e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right)^1 e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^A\right)^1 e\left(R_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2\right)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e(S_{\mathsf{sig}}^2, \mathsf{G}_2)^1 e(\mathsf{G}_1, T_{\mathsf{sig}}^2)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}} || \mathsf{ID}_{\mathsf{ctr}})\right)^1 e(\mathsf{G}_1, \sigma_{\mathsf{ctr}})^{-1} = 1_{\mathbb{G}_T} .$$

This PPE involves both relations in $\mathcal{L}_1.5$ and $\mathcal{L}_1.6$ and we make sure to use the same commitment to the group element $\mathsf{vk}_{\mathsf{sig}}$ [47].

G.1.2 Language $\mathcal{L}_2$. In the second relation of the proposed generic ul-PCS scheme, the prover takes the statement $x_{\mathsf{st}} := (\mathsf{ID}_S, \mathsf{ct}_R, \mathsf{vk}_{\mathsf{sig}}^A)$ and the witness $w_{\mathsf{st}} := (\mathsf{k}, \mathsf{ctr}, \mathsf{sk}_{f_x}, \sigma_{\mathsf{sig}}^2)$ as inputs and acts as follows:

$\mathcal{L}_2.1$. $\boxed{\mathsf{ID}_S = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})}$: We use the sigma protocols to demonstrate the well-formedness of DY PRF, by running $\Sigma\text{-PRF}\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_S = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\}$, as described in Figure 24.

$\mathcal{L}_2.2$. $\boxed{\mathsf{PE.Dec}(\mathsf{sk}_{f_x}, \mathsf{ct}_R) = 1}$: To prove the knowledge of an PE secret key $\mathsf{sk}_{f_x} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_N)$ and proving the fact that it correctly decrypts the receiver's PE ciphertext $\mathsf{ct}_R := (\mathsf{ct}_1, \ldots, \mathsf{ct}_N)$ to the identity value $1_{\mathbb{G}_T}$, we utilize the GS proof systems. As we already discussed in Appendix F.1 the OT12's decryption algorithm can be formalized with a PPE equation of the form, $\prod_{j=1}^N e\left(\mathsf{sk}_j, \mathsf{ct}_j\right) = e(\mathsf{G}_1, \mathsf{G}_2)$.

$\mathcal{L}_2.3$. $\boxed{\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{k}, \mathsf{sk}_{f_x}), \sigma_{\mathsf{sig}}^3) = 1}$: This signature is also instantiated by a SPS and similar to the previous languages, to prove the knowledge of a SPS signature σ_{sig}^3 on hidden message $\vec{M} = (\mathsf{G}_1^{\mathsf{k}}, \mathsf{sk}_{f_x})$ that is signed by the CA we can use the GS proof systems. Towards the arithmetization of this relation we can write the verification equation with a PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_0^A\right)\left(\prod_{j=1}^N e\left(\mathsf{sk}_j, \hat{X}_j^A\right)\right) = e\left(R_{\mathsf{sig}}^3, T_{\mathsf{sig}}^3\right) \wedge e(S_{\mathsf{sig}}^3, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^3)$, where $\mathsf{sk}_{f_x} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_N)$, $\mathsf{vk}_{\mathsf{sig}}^A := (\hat{X}_0^A, \hat{X}_1^A, \ldots, \hat{X}_N^A)$ and $\sigma_{\mathsf{sig}}^3 := (R_{\mathsf{sig}}^3, S_{\mathsf{sig}}^3, T_{\mathsf{sig}}^3)$.

♦ Additionally, the prover runs $\Sigma\text{-Bridging}\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$, where cm_1 is obtained via Σ-PRF protocol while the commitment cm_2 is computed by the GS proof systems on the knowledge of SPS signature σ_{sig}^3.

♦ To demonstrate the fact that the used PE's secret key sk_{f_x} in the relations discussed in $\mathcal{L}_2.2$ and $\mathcal{L}_2.3$ are the same, the prover makes a bridge between them. Due to the fact that both of these relations are proven via GS proof systems, we can instead combine them and prove the following PPE.

$$e(\mathsf{G}_1, \mathsf{G}_2)^{-1} \prod_{j=1}^N e\left(\mathsf{sk}_j, \mathsf{ct}_j\right)^1 = 1_{\mathbb{G}_T} \wedge$$
$$e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_0^A\right)^1 \prod_{j=1}^N e\left(\mathsf{sk}_j, \hat{X}_j^A\right)^1 e\left(R_{\mathsf{sig}}^3, T_{\mathsf{sig}}^3\right)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e(S_{\mathsf{sig}}^3, \mathsf{G}_2)^1 e(\mathsf{G}_1, T_{\mathsf{sig}}^3)^{-1} = 1_{\mathbb{G}_T} .$$

This PPE involves both relations in $\mathcal{L}_2.2$ and $\mathcal{L}_2.3$ using the same commitment to group element sk_{f_x}.

G.2 ul-PCS for Separable Policies

G.2.1 Language $\mathcal{L}_1$. The first language in the ul-PCS with separable policies takes the instance $x_{\mathsf{st}} := (T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ct}_{\mathsf{ctr}}, \mathsf{vk}_{\mathsf{sig}}^{A,R}, \mathsf{pk}_{\mathsf{PKE}}^A)$ and witness $w_{\mathsf{st}} := (\mathsf{k}, \mathsf{ctr}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, m_x, \sigma_{\mathsf{sig}}^1, \sigma_{\mathsf{ctr}})$ as inputs and then the prover proves the satisfiability of the following relations:

$\mathcal{L}_1.1$. $\boxed{\mathsf{ID}_{\mathsf{ctr}} = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})}$: To prove the well-formedness of the DY PRF, we use the sigma protocol described in Figure 24 and the prover runs $\Sigma\text{-PRF}\left\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_{\mathsf{ctr}} = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\right\}$.

$\mathcal{L}_1.2$. $\mathsf{ctr} < T_{\mathsf{Rand}}$: Additionally to prove $\mathsf{ctr} \in [0, T_{\mathsf{Rand}})$, the prover uses the range-proofs.

♦ To demonstrate the fact that the used ctr in the above proofs are identical, the prover utilizes the bridging sigma protocol described in Figure 22. More precisely, it runs $\Sigma\text{-Bridging}\{(\mathsf{ctr}, e_1, e_2, 0, 0) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{ctr}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{ctr}}\mathsf{H}_2^{e_2}\}$, where cm_1 is obtained via Σ-PRF protocol while the commtiment cm_2 is computed by the range-proof protocol.

$\mathcal{L}_1.3$. $\boxed{\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^{A,R}, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}, m_x), \sigma_{\mathsf{sig}}^1) = 1}$: To prove the knowledge of a valid SPS signature σ_{sig}^1 on message $\vec{M} = (\mathsf{G}_1^{\mathsf{k}}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{G}_1^{m_x})$ signed by the CA we can prove the satisfiability of the PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^{A,R}\right) e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^{A,R}\right) e\left(\mathsf{G}_1^{m_x}, \hat{X}_3^{A,R}\right) = e\left(R_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1\right) \wedge e(S_{\mathsf{sig}}^1, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^1)$, where $\mathsf{vk}_{\mathsf{sig}}^{A,R} := (\hat{X}_1^{A,R}, \hat{X}_2^{A,R}, \hat{X}_3^{A,R})$ and $\sigma_{\mathsf{sig}}^1 := (R_{\mathsf{sig}}^1, S_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1)$. Thus we use the GS proofs to instantiate this relation in zero-knowledge.

♦ To prove that the used k in the first relation and the obove relation are the same, the prover use the bridging sigma protocols described in Figure 22. More precisely, it runs $\Sigma\text{-Bridging}\{(\mathsf{k}, e_1, e_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\}$, where cm_1 is obtained via Σ-PRF protocol while cm_2 is computed by the GS proof systems.

$\mathcal{L}_1.4$. $\boxed{\mathsf{ct}_{\mathsf{ctr}} = \mathsf{PKE.Enc}(\mathsf{mpk}_{\mathsf{PKE}}^A, m_x)}$: To prove the knowledge of a valid ciphertext encrypting the hidden message m_x, we utilize the sigma protocol described in Figure 21 and the prover runs $\Sigma\text{-ElGamal}\{(m_x, r, e) \mid \mathsf{ct}_1 = \mathsf{G}_1^r \wedge \mathsf{ct}_2 = \mathsf{G}_1^{m_x}(\mathsf{pk}_{\mathsf{PKE}}^A)^r \wedge \mathsf{cm} = \mathsf{G}_1^{m_x}\mathsf{H}_1^e\}$.

♦ To prove the message m_x in the SPS relation and the obove relation are the same, the prover runs Σ-Bridging$\{(m_x, e_1, e_2, u_1, 0) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1}\mathsf{K}_1^{u_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\}$. In which the commitment cm_1 is obtained via the GS proof of SPS signature σ^1_{sig}, while cm_2 is computed by the sigma protocol Σ-ElGamal.

$\mathcal{L}_1$**.5**. $\boxed{\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}, (\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \mathsf{ID}_{\mathsf{ctr}}), \sigma_{\mathsf{ctr}}) = 1}$: The knowledge of a BLS signature on public message $m = (\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}||\mathsf{ID}_{\mathsf{ctr}})$ signed under the hidden signing key $\mathsf{sk}_{\mathsf{sig}}$ can be written as a PPE of the form, $e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}||\mathsf{ID}_{\mathsf{ctr}})\right) = e(\mathrm{G}_1, \sigma_{\mathsf{ctr}})$, where $H(\cdot)$ is a hash-to-curve function as a part of public parameters. We use the GS proofs to instantiate this relation in zero-knowledge.

♦ To show the fact that the verification key, $\mathsf{vk}_{\mathsf{sig}}$, used in the above GS proof is already certified by the CA and is identical to the one in the GS of the SPS signature σ^1_{sig}, the prover makes a bridge between the relations discussed in $\mathcal{L}_1$.3 and $\mathcal{L}_1$.5 via proving the following PPE instead with shared commitments [47].

$$
\begin{aligned}
&e\left(\mathrm{G}_1^{\mathsf{k}}, \hat{X}_1^{A,R}\right)^1 e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^{A,R}\right)^1 \\
&e\left(\mathrm{G}_1^{m_x}, \hat{X}_3^{A,R}\right)^1 e\left(R^1_{\mathsf{sig}}, T^1_{\mathsf{sig}}\right)^{-1} = 1_{\mathbb{G}_T} \wedge \\
&e(S^1_{\mathsf{sig}}, \mathrm{G}_2)^1 e(\mathrm{G}_1, T^1_{\mathsf{sig}})^{-1} = 1_{\mathbb{G}_T} \wedge \\
&e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}||\mathsf{ID}_{\mathsf{ctr}})\right)^1 e(\mathrm{G}_1, \sigma_{\mathsf{ctr}})^{-1} = 1_{\mathbb{G}_T}\,.
\end{aligned}
$$

This PPE involves both relations in $\mathcal{L}_1$.3 and $\mathcal{L}_1$.5 and we use the same commitment to the group element $\mathsf{vk}_{\mathsf{sig}}$ in all relations.

G.2.2 Language $\mathcal{L}_2$. The second language in the ul-PCS with separable policies takes the instance $x_{\mathsf{st}} = \left(\mathsf{ID}_S, \mathsf{ct}_R, \mathsf{vk}^{A,S}_{\mathsf{sig}}, \mathsf{pk}^A_{\mathsf{PKE}}\right)$ and witness $w_{\mathsf{st}} = \left(\mathsf{k}, \mathsf{ctr}, \mathsf{sk}^A_{\mathsf{PKE}}, \sigma^2_{\mathsf{sig}}\right)$ as inputs and then the prover proves the satisfiability of the following relations:

$\mathcal{L}_2$**.1**. $\boxed{\mathsf{ID}_S = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})}$: The prover runs Σ-PRF$\left\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_S = \mathrm{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\right\}$, depicting the well-formedness of ID_S.

$\mathcal{L}_2$**.2**. $\boxed{\mathsf{DS.Verify}(\mathsf{vk}^{A,R}_{\mathsf{sig}}, (\mathsf{k}, \mathsf{sk}^A_{\mathsf{PKE}}), \sigma^2_{\mathsf{sig}}) = 1}$: The possession of a SPS signature on message $\vec{M} = (\mathrm{G}_1^{\mathsf{k}}, \mathsf{sk}^A_{\mathsf{PKE}})$, signed by the CA can be written as a PPE of the form, $e\left(\mathrm{G}_1^{\mathsf{k}}, \hat{X}_1^{A,S}\right) e\left(\mathrm{G}_1^{\mathsf{sk}^A_{\mathsf{PKE}}}, \hat{X}_2^{A,S}\right) = e\left(R^2_{\mathsf{sig}}, T^2_{\mathsf{sig}}\right) \wedge e(S^2_{\mathsf{sig}}, \mathrm{G}_2) = e(\mathrm{G}_1, T^2_{\mathsf{sig}})$, where $\mathsf{vk}^{A,S}_{\mathsf{sig}} := (\hat{X}_1^{A,S}, \hat{X}_2^{A,S})$ and $\sigma^2_{\mathsf{sig}} := (R^2_{\mathsf{sig}}, S^2_{\mathsf{sig}}, T^2_{\mathsf{sig}})$. We use the GS proof systems to represent the satisfiability of this PPE.

♦ To demonstrate the fact that the PRF key k used in Σ.PRF and in the above GS proof is indeed the same, the prover runs Σ-Bridging$\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathrm{G}_1^{\mathsf{k}}\mathrm{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathrm{G}_2^{\mathsf{k}}\mathrm{H}_2^{e_2}\mathrm{K}_2^{u_2}\}$. The commitment cm_1 is obtained via the sigma protocol described in Figure 24 while the commitment cm_2 is computed by the GS proof of knowledge σ^2_{sig} on the satisfiability of the SPS.

$\mathcal{L}_2$**.3**. $\boxed{\mathsf{PKE.Dec}(\mathsf{sk}^A_{\mathsf{PKE}}, \mathsf{ct}_R) = 1}$: The knowledge of a valid secret key $\mathsf{sk}^A_{\mathsf{PKE}}$ such that it can decrypt the receiver's ciphertext ct_R to $m = 1$. To prove this relation in zero-knowledge we use the Dlog sigma protocol described in Figure 20 and the prover runs, Σ-Dlog $\left\{(\mathsf{sk}^A_{\mathsf{PKE}}) \mid \mathsf{ct}_{R,2}/\mathrm{G}_1 = (\mathsf{ct}_{R,1})^{\mathsf{sk}^A_{\mathsf{PKE}}}\right\}$.

♦ To show the used secret key $\mathsf{sk}^A_{\mathsf{PKE}}$ in the above sigma protocol is the same as the one signed in σ^2_{sig}, the prover runs Σ-Bridging$\{(\mathsf{sk}^A_{\mathsf{PKE}}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathrm{G}_1^{\mathsf{sk}^A_{\mathsf{PKE}}}\mathrm{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathrm{G}_2^{\mathsf{sk}^A_{\mathsf{PKE}}}\mathrm{H}_2^{e_2}\mathrm{K}_2^{u_2}\}$, where cm_1 is obtained via Σ-Dlog protocol while the commitment cm_2 is computed by the GS proof systems on the knowledge of SPS signature σ^2_{sig}.

$\mathcal{L}_2$**.4**. $\boxed{\mathsf{sk}^A_{\mathsf{PKE}} \approx \mathsf{vk}^A_{\mathsf{PKE}}}$: The prover to show the knowledge of the secret key $\mathsf{sk}^A_{\mathsf{PKE}}$ and the fact that it corresponds to the public encryption key $\mathsf{pk}^A_{\mathsf{PKE}}$, uses the sigma protocol described in Figure 20 and runs Σ-Dlog $\left\{(\mathsf{sk}^A_{\mathsf{PKE}}) \mid \mathsf{pk}^A_{\mathsf{PKE}} = \mathrm{G}_1^{\mathsf{sk}^A_{\mathsf{PKE}}}\right\}$.

♦ The prover runs the bridging sigma protocol Σ-Bridging$\{(\mathsf{sk}^A_{\mathsf{PKE}}, e_1, e_2, 0, 0) \mid \mathsf{cm}_1 = \mathrm{G}_1^{\mathsf{sk}^A_{\mathsf{PKE}}}\mathrm{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathrm{G}_2^{\mathsf{sk}^A_{\mathsf{PKE}}}\mathrm{H}_2^{e_2}\}$ to prove the used secret key $\mathsf{sk}^A_{\mathsf{PKE}}$ in the above relations are the same. The commitment cm_1 is obtained by the sigma protocol Σ-Dlog of the PKE's decryption correctness while the commitment cm_2 is computed by sigma protocol Σ-Dlog to show the relation of PKE's public and secret keys.

G.3 ul-PCS for RBAC Policies

G.3.1 Language $\mathcal{L}_1$. The first language in the RBAC ul-PCS takes the instance $x_{\mathsf{st}} = \left(T_{\mathsf{Rand}}, \mathsf{ID}_{\mathsf{ctr}}, \mathsf{vk}^{\mathsf{ctr}}_{\mathsf{sig}}, \vec{M} := (\mathrm{A}'_1, \mathrm{A}'_2, \mathrm{G}'_2), \mathsf{vk}^A_{\mathsf{sig}}\right)$ and witness $w_{\mathsf{st}} = \left(\mathsf{k}, \mathsf{ctr}, \mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}}, w_{\mathsf{k}}, \sigma^1_{\mathsf{sig}}, \sigma_{\mathsf{sig}}\right)$ as inputs and the prover proves the satisfiability of the following relations:

$\mathcal{L}_1$**.1**. $\boxed{\mathsf{ACC.MemVrf}(\mathrm{A}'_1, \mathsf{k}, w_{\mathsf{k}}) = 1}$: To prove the possession of a hidden membership witness w_{k} that verifies the accumulator value A'_1 the prover uses the GS proof systems. The satisfiability of the verification of the given accumulator scheme can be written as a PPE of the form, $e\left(w_{\mathsf{k}}, \mathrm{A}'_1\right) e\left(w_{\mathsf{k}}, (\mathrm{G}'_2)^{\mathsf{k}}\right) = e\left(\mathrm{G}_1, \mathrm{G}'_2\right)$. We use the GS proofs to prove the satisfiability of this equation in zero-knowledge.

$\mathcal{L}_1$**.2**. $\boxed{\mathsf{ID}_{\mathsf{ctr}} = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})}$: We use the sigma protocol described in Figure 24 to prove the well-formedness of DY PRF, i.e. Σ-PRF$\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_{\mathsf{ctr}} = \mathrm{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\}$ over cyclic group $\mathbb{G}_1$.

♦ The prover to make a bridging between the above relations and showing the fact that the used PRF key k in the both of them is the same secret witnees runs

Σ-Bridging$\{(\mathsf{ctr}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{ctr}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{ctr}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$. In which the commitment cm_1 is obtained via Σ-PRF protocol while the commtiment cm_2 is computed in the GS proof on the satisfiability of the accumulator verification algorithm.

$\mathcal{L}_1$.3. $\mathsf{ctr} < T_{\mathsf{Rand}}$: Additionally, the prover utilizes the range-proof techniques to prove $\mathsf{ctr} \in [0, T_{\mathsf{Rand}})$.

♦ The prover runs Σ-Bridging$\{(\mathsf{ctr}, e_1, e_2, 0, 0) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{ctr}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{ctr}}\mathsf{H}_2^{e_2}\}$ to prove the used hidden counter ctr in the above relations is the same. In which the commitment cm_1 is obtained via Σ-PRF protocol while the commtiment cm_2 is computed in the range-proof protocol.

$\mathcal{L}_1$.4. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{k}, \mathsf{vk}_{\mathsf{sig}}), \sigma_{\mathsf{sig}}^1) = 1$: To prove the verification phase of the SPS signature σ_{sig}^1 satisfies under message $\vec{M} = (\mathsf{G}_1^{\mathsf{k}}, \mathsf{vk}_{\mathsf{sig}})$ and the fact that it is signed by the CA, we can show it via a PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right) e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^A\right) = e\left(R_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1\right) \wedge e(S_{\mathsf{sig}}^1, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^1)$, where $\mathsf{vk}_{\mathsf{sig}}^A := (\hat{X}_1^A, \hat{X}_2^A)$ and $\sigma_{\mathsf{sig}}^1 := (R_{\mathsf{sig}}^1, S_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1)$. We use GS proof systems to show the satisfiability of this equation.

♦ To demonstrate that the same k in the first relation and the above relation is used, the prover makes a bridge between them by running Σ-Bridging$\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$, where the commitment cm_1 is obtained via Σ-PRF protocol while the commitment cm_2 is computed by the GS proof system on the validity of σ_{sig}^1.

$\mathcal{L}_1$.5. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}, (\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}}, \mathsf{ID}_{\mathsf{ctr}}), \sigma_{\mathsf{ctr}}) = 1$: To validate a newly generated verification key and to bind it with the new identifier $\mathsf{ID}_{\mathsf{ctr}}$, the prover needs to prove the satisfiability of a PPE relation described as, $e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}} || \mathsf{ID}_{\mathsf{ctr}})\right) = e(\mathsf{G}_1, \sigma_{\mathsf{ctr}})$ that represents the validity of BLS signature. We use GS proof systems to instantiate this relation in zero-knowledge.

♦ To show the fact that the verification key, $\mathsf{vk}_{\mathsf{sig}}$, used in the above GS proof is already certified by the CA and is identical to the one in the GS of the SPS signature σ_{sig}^1, the prover makes a bridge between the relations discussed in $\mathcal{L}_1$.4 and $\mathcal{L}_1$.5 via proving the following PPE instead.

$$e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right)^1 e\left(\mathsf{vk}_{\mathsf{sig}}, \hat{X}_2^A\right)^1 e\left(R_{\mathsf{sig}}^1, T_{\mathsf{sig}}^1\right)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e(S_{\mathsf{sig}}^1, \mathsf{G}_2)^1 e(\mathsf{G}_1, T_{\mathsf{sig}}^1)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e\left(\mathsf{vk}_{\mathsf{sig}}, H(\mathsf{vk}_{\mathsf{sig}}^{\mathsf{ctr}} || \mathsf{ID}_{\mathsf{ctr}})\right)^1 e(\mathsf{G}_1, \sigma_{\mathsf{ctr}})^{-1} = 1_{\mathbb{G}_T} .$$

This PPE involves both relations in $\mathcal{L}_1$.4 and $\mathcal{L}_1$.5 with a single commitment to $\mathsf{vk}_{\mathsf{sig}}$.

G.3.2 Language $\mathcal{L}_2$. In the second relation, the prover takes the instance $x_{\mathsf{st}} = (\mathsf{ID}_S, \mathsf{ct}_R, \mathsf{vk}_{\mathsf{sig}}^A, \mathsf{pp}', \mathsf{A}')$ and the witness $w_{\mathsf{st}} = (\mathsf{k}, \mathsf{ctr}, x, w, \sigma_{\mathsf{sig}}^2)$ as input and acts as follows:

$\mathcal{L}_2$.1. $\mathsf{ID}_S = \mathsf{PRF.Eval}(\mathsf{k}, \mathsf{ctr})$: To prove the well-formedness of the PRF evaluation the prover runs the sigma protocol Σ-PRF$\{(\mathsf{k}, \mathsf{ctr}) \mid \mathsf{ID}_S = \mathsf{G}_1^{1/(\mathsf{k}+\mathsf{ctr})}\}$ over the cyclic group $\mathbb{G}_1$.

$\mathcal{L}_2$.2. $\mathsf{DS.Verify}(\mathsf{vk}_{\mathsf{sig}}^A, (\mathsf{k}, w), \sigma_{\mathsf{sig}}^2) = 1$: This relation can be formulated by a PPE of the form, $e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right) e\left(w, \hat{X}_2^A\right) = e\left(R_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2\right) \wedge e(S_{\mathsf{sig}}^2, \mathsf{G}_2) = e(\mathsf{G}_1, T_{\mathsf{sig}}^2)$, where $\mathsf{vk}_{\mathsf{sig}}^A := (\hat{X}_1^A, \hat{X}_2^A)$ and $\sigma_{\mathsf{sig}}^2 := (R_{\mathsf{sig}}^2, S_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2)$ and the prover can prove the satisfiability of the relation by GS proof systems.

♦ The prover runs Σ-Bridging$\{(\mathsf{k}, e_1, e_2, 0, u_2) \mid \mathsf{cm}_1 = \mathsf{G}_1^{\mathsf{k}}\mathsf{H}_1^{e_1} \wedge \mathsf{cm}_2 = \mathsf{G}_2^{\mathsf{k}}\mathsf{H}_2^{e_2}\mathsf{K}_2^{u_2}\}$ to prove the fact that the PRF key k used in Σ.PRF and is already signed by the CA. The commitment cm_1 is obtained via the sigma protocol described in Figure 24 while the commitment cm_2 is computed by the GS proof of knowledge σ_{sig}^2.

$\mathcal{L}_2$.3. $\mathsf{ACC.MemVrf}(\mathsf{A}', x, w) = 1$: Similar to the previous languages, the prover can describe the membership verification of the accumulator scheme by the satisfiability of a PPE of the form, $e(w, \mathsf{A}')\, e\left(w, (\mathsf{G}_2')^x\right) = e\left(\mathsf{G}_1, \mathsf{G}_2'\right)$. Thus it runs the GS proof to show the possession of hidden parameters.

♦ The prover bridges the relations describe in $\mathcal{L}_2$.2 and $\mathcal{L}_2$.3 to show the fact that the membership witness w which passes the accumulator verification is already certifies and is signed in SPS signature σ_{sig}^2. For this aim the prover proves the following PPE instead:

$$e\left(\mathsf{G}_1^{\mathsf{k}}, \hat{X}_1^A\right)^1 e\left(w, \hat{X}_2^A\right)^1 e\left(R_{\mathsf{sig}}^2, T_{\mathsf{sig}}^2\right)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e(S_{\mathsf{sig}}^2, \mathsf{G}_2)^1 e(\mathsf{G}_1, T_{\mathsf{sig}}^2)^{-1} = 1_{\mathbb{G}_T} \wedge$$
$$e\left(w, \mathsf{A}'\right)^1 e\left(w, (\mathsf{G}_2')^x\right)^1 e\left(\mathsf{G}_1, \mathsf{G}_2'\right)^{-1} = 1_{\mathbb{G}_T} .$$

H DISTRIBUTED SETUP AND KEYGEN ALGORITHMS

In the following, we showcase that using standard techniques we can achieve distributed implementations of the algorithms Setup and KeyGen for our three constructions. We assume an honest-but-curious model for the sake of the argument, however a lifting to malicious security would again follow standard techniques.

H.1 The Generic ul-PCS Scheme

First, we look at how the generic ul-PCS, proposed in Figure 5, and its concrete instantiation based on the OT12's inner product predicate encryption [57] can be distributed. Recall that the CA holds the predicate encryption's master secret key, $\mathsf{msk}_{\mathsf{PE}}$, along with a signature key $\mathsf{sk}_{\mathsf{sig}}^A$. On the other hand, a user keeps will obtain the PRF seed k and a predicate encryption secret key sk_f along with its root signature key-pair $(\mathsf{sk}_{\mathsf{sig}}, \mathsf{vk}_{\mathsf{sig}})$. To generate these secret elements in a distributed manner, we can follow the following steps:

(1) *CA-side setup:*

(a) Given the description of the OT12 IP-PE scheme in Appendix F, we can generate the keys in a distributed way, where each server holds a share B_i^* of the matrix $B^* = \prod_i B_i^*$ (component-wise product). This could be done with a standard MPC, and essentially, we need a sum-sharing of a matrix X and its inverse Y. In particular, this means each entry Y_{ij} is shared among n distinct certificate authorities. While this is a heavier computation, implementations of such an operation based on the methods by Blom et al. are possible [13].
(b) Each CA possesses its own signature key-pair.

(2) *Registration of a client for attributes x:*
(a) Each CA samples random seed k_i and a public key share vk_i.
(b) The CAs create additional shared randomness in anticipation of the creation of the secret functional key. They compute a sum-sharing of $n+1$ random elements r_k.
(c) Then they run a multiplication protocol to obtain a sharing of selected matrix elements: $r_1 Y_{i,2}, \ldots, r_1 Y_{i,n+1}$ (those are the positions for generating a scaled vector of attributes) and $r_j Y_{i,3n+j} (1 < j \leq n+1)$ (those are the positions where random exponents are needed).
(d) Recalling from Appendix F.1 that in OT12 the functional key sk_f is a vector whose ith component is $\prod_{j=1}^{N} \mathrm{G}_1^{Y_{ij} z_j}$, we observe that each CA can compute a meaningful share sk_f^i by doing this computation based on the attribute x and the sharing of elements Y_{ij} resp. $r_k Y_{ij}$ (for those indices where additional randomness is needed).
(e) The CA signs the pairs (k_i, x), $(\mathsf{k}, \mathsf{sk}_f^i)$, and $(\mathsf{k}, \mathsf{vk}_i)$.
(f) *Aggregation step:*
(i) Functional key shares are aggregated by component-wise multiplication of the vectors sk_f^i — The addition in the exponent leads to the expression in (d) as everything has been computed as a sum-sharing.
(ii) Seed shares are summed up $\mathsf{k} = \sum_i \mathsf{k}_i$.
(iii) Root key pairs are summed up as well (e.g. assuming a simple DL-based signature scheme).

(3) Finally, the aggregated pairs $(\mathsf{k}, \mathsf{sk}_f)$, (k, x) and $(\mathsf{k}, \mathsf{vk}_{\mathsf{sig}})$ are certifiable, because in any of the languages $\mathcal{L}_1$ and $\mathcal{L}_2$, instead of proving the knowledge of a signature from the CA, one would have to prove the aggregation is done correctly based on the signed shares by each CA. While conceptually possible, by an additional round of interaction, one can even shift more computational overhead to the registration phase as outlined below:

3'. Alternatively to the above certification, one can add one round of interaction, where the client commits to each aggregated pair, proves the well-formedness using a NIZK and obtains a threshold signature on the commitments. In this case, each pair is certifiable in the first and second NIZK languages by having one additional commitment plus a signature on it. In this case, for further efficiency, the utilized SPS scheme can be replaced with the recent Threshold SPS-scheme of Crites et al. [29]. In this case, each CA has its own SPS signature key-pair and a sufficiently large number of issuers is needed to obtain a valid signature (with respect to the aggregated public key). This strategy pushes most of the computational overhead into the registration phase.

Furthermore, simplifications can be made depending on the adversarial model. We observe that the root key pair $(\mathsf{sk}_{\mathsf{sig}}, \mathsf{vk}_{\mathsf{sig}})$ for the party is never revealed by the party in any operation, and thus we can simply let one of the servers decide for that one if we are in an honest-but-curious setting.

H.2 The ul-PCS with Separable Policies

Similarly, we can distribute the generation of the secret keys in the ul-PCS scheme with separable policies. In this scheme, the PE scheme is "realized" using ordinary PKE with keys $(\mathsf{pk}_{\mathsf{PKE}}, \mathsf{sk}_{\mathsf{PKE}})$. Furthermore, we have signature keys to authorize sender and receiver predicates. Compared to the generic scheme discussed above, it is much simpler and we can run a distributed-key generation in advance and each CA has its own signature key-pair. We briefly discuss how the user's registration works concretely.

(1) *CA-side setup:* Each CA samples random PRF seed $\mathsf{k}_i \xleftarrow{\$} \mathbb{Z}_p^*$ and a public key share vk_i.
(2) *Registration of a client for attributes x:* Each CA issues a signature on the value $(\mathsf{k}_i, \mathsf{vk}_i, m)$, where $m \in \{0,1\}$ is a bit. If $m = 1$, the client also gets a signature on $(\mathsf{k}_i, \mathsf{sk}_i)$.
The client performs standard aggregation to compute all relevant values ($\mathsf{sk}_{\mathsf{sig}}$ from all sk_i's, k from all k_i's).
(3) Certification is again possible via a NIZK, or via one more round of interaction as above.

H.3 The Role-based ul-PCS Construction

The signing process in the role-based ul-PCS is as above, but the relevant values that could break privacy are the accumulator witnesses (because they would allow to test which attributes can send to a target public key), and the seed values. Hence, here one has to do the following:

(1) CA setup: As the accumulator witnesses in this case are just signatures on roles that belong to an accumulator value, we just set up a threshold signature scheme. Each CA then holds a signature share on a role i for accumulator A (identified by the signature public key).
(2) Registration of a client for attributes x:
(a) Each CA samples random PRF seed k_i and a public key share vk_i.
(b) The user can simply obtain the partial signature shares and a combination of them and finally is in possession of the full witness for its role x.
(c) The remaining steps are as above: the client can reconstruct the full seed k_i, the full root signature keypair $(\mathsf{vk}_{\mathsf{sig}}, \mathsf{sk}_{\mathsf{sig}})$, and has all witnesses.
(3) Certification can be done via a NIZK or via another round of interaction as above.

I DETAILS OF SECTION 6

I.1 Preliminaries on One-Time Accounts (OTA)

An OTA scheme [33] is defined as a tuple of algorithms $\mathsf{OTA} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{NoteGen}, \mathsf{Enc}, \mathsf{Receive}, \mathsf{NulEval})$ with the following syntax and intended semantics:

- Setup: Generates the public parameters that is given implicitly to any algorithm below as input.
- KeyGen: Generates an asymmetric key-pair $(\mathsf{pk}, \mathsf{sk})$.
- NoteGen$(\mathsf{pk}, \vec{a}; r)$: Takes a public key and a vector of type-value pairs and generates the note, i.e. the account.
- Enc$(\mathsf{pk}, (\vec{a}, r))$: Encrypts the information toward the recipient such that the recipient will be able to reconstruct the note's content and to spend it (see below) .
- Receive$(\mathsf{note}, C, \mathsf{sk})$: If the note and ciphertext are created for the public key belonging to sk, then the algorithm returns the values $(\vec{a}, r)$, and otherwise returns $\perp$.
- NulEval(sk, r): Returns the nullifier value that is tied to a particular note (generated with randomness r). The nullifier is needed to spend the tokens contained in a note.

OTA's must be accompanied by some efficient NIZK languages, including the ones we need in our construction in Section 6, which are shown to be efficiently realizable [33] using for example Groth-Sahai proof systems. The security requirements from an OTA scheme include: (1) Nullifiers should appear pseudo-random and be unique, such that they can be presented as evidence of spending a coin, and double spends would directly visible by repeated nullifiers, (2) the note is binding to the key and values, unique, as well as private in that it hides its content. We discuss these requirements in the security analysis of our extended scheme.

I.2 Details on Construction I

Construction I. The idea is to use the PCS scheme to sign the note but hide the involved public keys and signatures inside the OTA ciphertext. The owner of a note can then use the relevant values in a zero-knowledge proof of knowledge when claiming, as described above, a note as part of a transaction.

This generic composition is simple and obviously preserves all underlying OTA guarantees (cf. Appendix I), but pushes a lot of complexity into the NIZK. In order to show that the nullifier nul spends a note note (which contains a vector of type-value pairs $\vec{a}$ and is generated with randomness r that is contained in the ledger state st, at least the following language must be supported for the construction:

$$L = \{(\mathsf{mpk}, st, \mathsf{nul}) \mid \exists(\mathsf{note}, \mathsf{sk}_{\mathsf{ota}}, \vec{a}, r, \mathsf{pk}^S_{\mathsf{pcs}}, \mathsf{pk}^R_{\mathsf{pcs}}, \mathsf{sk}^R_{\mathsf{pcs}}, \sigma_{\mathsf{pcs}}) : \mathsf{note} \in st \wedge \mathsf{note} = \mathsf{NoteGen}(P(\mathsf{sk}_{\mathsf{ota}}), \vec{a}, r) \wedge \mathsf{nul} = \mathsf{NulEval}(\mathsf{sk}_{\mathsf{ota}}, r) \wedge \mathsf{Verify}(\mathsf{mpk}, \mathsf{pk}^S_{\mathsf{pcs}}, pk^R_{\mathsf{pcs}}, \mathsf{note}, \sigma_{\mathsf{pcs}}) \wedge \mathsf{pk}^R_{\mathsf{pcs}} = P(\mathsf{sk}^R_{\mathsf{pcs}})\},$$

where $P(\mathsf{sk})$ is an assumed mapping that computes the public key from the secret key (e.g., to prove knowledge of the secret key).

I.3 Security Analysis of Construction II

In this section, we elaborate on the provided guarantees of our construction presented in Section 6. Unlike the strawman approach, which is easily seen to be as secure as OTA, for our more efficient construction, we trade some security for efficiency. We now elaborate on the security provided by that construction following the OTA security goals.

I.3.1 Soundness and binding. An OTA ciphertext should decrypt to values that would correctly reconstruct the note that was given to it. On the other hand, binding ensures that a note is essentially a binding commitment to the vector $\vec{a}$. Both of these properties are satisfied by the above construction since we do not interfere with the generation of the OTA note.

I.3.2 Note and Ciphertext Privacy. Privacy mandates note and ciphertext hiding as well as note and encryption anonymity. If the underlying OTA scheme satisfies this, then the above construction trivially achieves it too. This is due to the fact that we do not interfere with note generation and that all the additional values are hidden by encrypting them using the underlying OTA encryption procedure.

I.3.3 Note Uniqueness. Note uniqueness captures that honestly generated notes (aka addresses) do not collide, except with negligible probability. This is obviously fulfilled by our construction.

I.3.4 Nullifier Uniqueness and collision resistance. Nullifier uniqueness demands that for the same note, no two nullifiers can be constructed and that the probability of two nullifiers colliding is negligible. As above, this is retained by the construction if the underlying OTA scheme satisfies it.

I.3.5 Nullifier security. The most crucial change of our construction is the nullifier. We gain efficiency by including signatures and (re-randomized) keys as part of the nullifier, but we trade the strong pseudo-random property, which has some security implications (compared to the strawman approach).

If the creator of a note is honest, the corresponding owner is able to spend the note in a private and anonymous way. In particular, if the PCS recipient key is re-randomized accordingly, no linking within the transaction log is possible thanks to the hiding and unlinkability property of PCS and the security of the underlying OTA scheme. The transaction log only reveals that parties are transacting which are allowed to transact by the policy. On the other hand, if different notes are created for the same PCS receiver key, then the only information that leaks from this, is the fact that the same party must, again, be transacting—but no link exists to the actual note or other transactions that use a re-randomized receiver key of this party, thanks to the privacy of the commitment scheme, the unlinkability of the sender PCS key (which by default gets re-randomized), and the security of the underlying OTA scheme.

However, if the creator of a note is malicious, then this creator (and only this creator) has enough information to determine that the owner of the note has been spending the note in a transaction. This is due to the presence of the additional PCS-related values that are revealed (as part of the nullifier). It is however possible to remedy this situation proactively, namely by spending the note to itself using a freshly randomized PCS key as soon as the transaction appears in the log. This is incidentally one of the recommended measures by Zcash to achieve *everlasting anonymity* [12].

Finally, we observe that spending a note is only possible if a party has access to the OTA private key and the PCS private key (which follows from the security provided by the underlying OTA nullifier and the unforgeability of the PCS signature on this nullifier).